Problem Set #6: Liquid and Solid Rocket Propulsion

- Homework solutions should be neat and logically presented, see format requirements at http://seitzman.gatech.edu/classes/ae4451/homeworkformat.html.
- If appropriate, include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**.
- Always indicate any **assumptions** you make. If you use any results or equations from the class notes or text in your solutions, please note and **reference** them (but you better be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. If possible, only use numbers/values in the final steps of each solution.

1. Methane/Oxygen LRE

The schematic shows a liquid rocket engine cycle, for a medium thrust launch vehicle, intended to produce a thrust of 896 kN (~200klb_f) at standard sea-level operation. The fuel is liquid methane (LCH₄) and the oxidizer is liquid oxygen (LOX), with the rocket is designed to run with an overall oxidizer-fuel ratio ($\dot{m}_{O_2}/\dot{m}_{CH_4}$) of 3.20.

The following conditions are known: LOX is stored at T₁=85.0 K, p₁=3.90 bar and LCH₄ is stored at T₄=106 K, p₄=3.10 bar. The main combustion chamber will operate at a pressure of 124 bar, with a stagnation temperature at the exit of the main combustor, estimated from the overall O/F ratio, to be 3607 K. Furthermore, the pre-combustor is designed to operate at an O/F ratio of 0.500, producing an exit stagnation temperature of 1123K. The turbine's adiabatic efficiency is expected to be 71.0%, while the adiabatic efficiency of each pump should be 73.0%. The rocket's convergingdiverging nozzle will be perfectly expanded and expected to have a 96.0% adiabatic efficiency.



In this cycle, the fuel is used to cool the nozzle and combustion chamber (i.e., the thrust chamber assembly TCA), with the fuel vaporizing and being heated to $T_{o6}=245$ K. Based on typical designs, the pressure drop across the liquid oxidizer injectors should be 21% of the corresponding combustion chamber pressure, e.g., $p_{o2}=1.21 \times p_{o7}$. The injector pressure drop for the (gaseous) fuel and pre-combustor exhaust gases will be lower, 14% of the corresponding combustion chamber pressure. In addition, an orifice plate will be used to reduce the oxidizer pressure (between stations 2 and 3) in order to produce the correct pressure before the main combustion chamber's injector. Finally, we expect the pressure drop across the TCA cooling jacket to be 15.0% of the cooling jacket exit pressure (i.e., $p_{o5}=1.15 \times p_{o6}$).

Assume there are no pressure losses within the combustion chambers (e.g., $p_{09}=p_{010}$), no pressure losses in any of the other flow lines, and that all the <u>gases</u> are thermally

and calorically perfect. If you need properties of the unburned reactants (liquid and gaseous) or of the combustion products (from either of the combustion chambers), use values **only** from the table below.

	ρ (kg/m³)	<i>c_p</i> (J/kg⋅K)	Ŵ	γ
Liquid O ₂	1170	1670	32.00	
Liquid CH₄	440.	3500	16.04	
Gaseous O ₂			32.00	1.32
Gaseous CH₄			16.04	1.27
Pre-combustor Products			15.40	1.24
Main Combustor Products			21.10	1.17

- a) Find the gas temperature exiting the nozzle (T_e) and the specific impulse of this rocket engine. In your calculation, you may neglect the heat losses from the nozzle (i.e., assume the heat losses in the nozzle are negligible with respect to the kinetic energy leaving the nozzle).
- b) Determine the required mass flowrates for each of the propellants (methane and oxygen)
- c) Find the required oxidizer and fuel pump exhaust stagnation pressures (p_{02} and p_{05}), the turbine exhaust conditions (p_{08} and T_{08}), and the shaft power produced by the turbine.

2. Missile SRM

A solid rocket motor (SRM) for a missile is being designed to operate with a propellant composed of aluminized HTPB/AP at an altitude where the ambient pressure is 0.488 atm. The motor is required to produce a steady thrust of 19.50 kN (~4.4 klbf) for 6.00 sec. The propellant has a solid density of 1820. kg/m³ and a burning (regression) rate that can be approximated by the expression, $\dot{r} = 0.1960 \ p^{0.510} \ mm/s$, where the pressure *p* is in units of psia. The design pressure for the rocket's combustion chamber is 1350. psia, and at this pressure, a chemical equilibrium calculation suggests the combustion products have the following properties: adiabatic flame temperature of 1380. K, molecular weight of 24.1, and specific heat ratio of 1.240. Furthermore, the motor's nozzle is to have an expansion ratio of 12.8. You may assume the internal nozzle flow is quasi-1D, adiabatic and reversible.

- a) Find the minimum propellant mass that will need to be loaded into the motor to meet these requirements.
- b) Determine the throat diameter and burning surface area required to meet the above requirements.
- c) You have been asked to modify the design to create a dual thrust version, with an initial 1.00 sec of operation at a thrust level that is 2.5 times higher than the original (base) thrust, but without changing the throat diameter, nozzle expansion ratio or propellant composition. How much additional propellant (kg) would be required, and what be the burning surface area for the additional propellant? You may assume the combustion product properties remain the same.

Extra Credit.

Your heat transfer analysis team has been asked to help with the design of a regenerative cooling system for a TCA in a methane/oxygen rocket, where the liquid methane will be used to cool the walls of the TCA.

The TCA designers want you to focus on a location near the nozzle throat, where they have provided the following estimates for the hot combustion product flow:

T(K)	ρ (kg/m³)	c _p (kJ/kgK)	k(W/mK)	μ (kg/m/s)	u(m/s)	Diameter D(m)
3290.	2.90	2.42	0.430	1.11×10 ⁻⁴	1120	0.231

and the following properties for the liquid methane flow through the coolant channels:

T(K)	ρ (kg/m³)	c _p (kJ/kgK)	k(W/mK)	μ (kg/m/s)	u(m/s)	Hydraulic D _h (mm)
110.	440.	3.44	0.190	1.31×10 ⁻⁴	22.0	1.90

The engine designers have chosen to use a high conductivity, Cu-based alloy for the material of the wall between the hot combustion products and the liquid methane coolant. The properties of the alloy are given in the following table.

ρ (kg/m³)	c _p (kJ/kgK)	k (W/mK)	
8980	0.381	354	

Based on a preliminary analysis, the team's structural engineers have indicated that the required wall thickness (t_w) for this alloy (to maintain a reasonable safety factor) will vary with the maximum wall temperature (T_{max}) according to the following relation (valid from 273K< T_{max} <900K)

 $t_w = 0.950 \text{ mm} - 1.22 \times 10^{-6} \text{ mm/K}^2 (T_{max} - 200. \text{ K})^2$

In addition, your team has identified the following Nusselt number heat transfer correlation as being appropriate for the conditions above (with $Nu = C_D Re_D^m Pr^n$):

Flow	C_D	m	п
Cold	0.0238	0.80	0.33
Hot	0.0213	0.80	0.39

with *D* defined as the TCA diameter for the hot flow and the hydraulic diameter of the coolant channels for the cold flow.

You have been tasked to determine the required wall thickness, along with the corresponding wall temperatures on the hot-gas side and coolant side of the TCA wall, as well as the heat flux through the wall. You may assume the TCA geometry can be reasonably modeled using a one-dimensional heat transfer analysis.