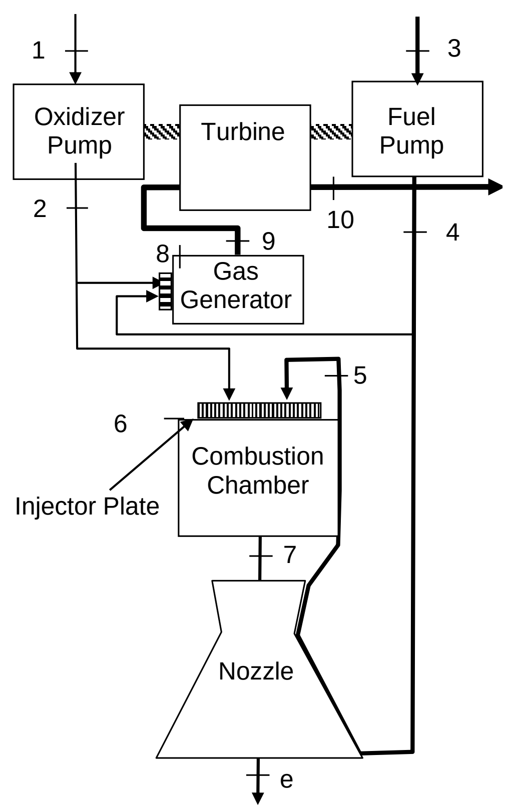
AE4451
Seitzman
Summer 2024

Problem Set #6: Liquid and Solid Rocket Propulsion

- Homework solutions should be neat and logically presented, see format requirements at http://seitzman.gatech.edu/classes/ae4451/homeworkformat.html.
- If appropriate, include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**.
- Always indicate any **assumptions** you make. If you use any results or equations from the class notes or text in you solutions, please note and **reference** them (but you better be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. If possible, only use numbers/values in the final steps of each solution.

1. Methane/Oxygen Rocket

A rocket engine for a launch vehicle is being designed using liquid methane (LCH_4) and oxygen (LOX) in a gas generator cycle (see figure). Each rocket engine on the vehicle needs to produce 445 kN (~100,000 lb_f) of thrust at a sea-level lift-off condition. The designers have chosen to operate with the following main combustion chamber conditions: an oxidizer-to-fuel mass ratio (O/F) of 3.10 and a stagnation pressure of 19.8 MPa. A single turbine will run two pumps (one for the oxidizer and one for the fuel). To produce a maximum allowable turbine inlet temperature of 1200. K, the gas generator will operate at an O/F of 0.550. In this design, most of fuel is also used to cool the nozzle and combustion chamber (i.e., the thrust chamber assembly TCA) before entering the main combustion chamber; thus the fuel entering the combustion chamber is already vaporized. The nozzle design is expected to produce an equivalent velocity of 3112 m/s.



Additionally, the LOX is stored at T_1=85.0 K, p_1=3.00 bar, while the LCH_4 is stored at T_3=106 K, p_3=3.10 bar. Furthermore, the turbine's adiabatic efficiency should be 62.0%, while the expected adiabatic efficiency of each of the pumps is 74.0%.

You can expect that the stagnation pressure drop across the liquid oxidizer injector for the main combustion chamber is 21% of the combustion chamber stagnation pressure, i.e., $p_{o2}=1.21\times p_{o6}$. The stagnation pressure drop for the liquid oxidizer injector in the gas generator will be 26% (of the gas generator chamber p_{o8}). Because gaseous fuel will be injected into the main combustion chamber, its injector stagnation pressure drop for the main combustion chamber will be lower, just 14%. However, the fuel cooling loop will also require an additional 20% stagnation pressure drop ($p_{o4}=1.20\times p_{o5}$). The liquid fuel injector for the gas generator will have a stagnation pressure sufficient such that its post-injection pressure matches the post-injection pressure of the oxidizer injector.

You should assume the stagnation pressure at the exit of a combustor is the same as the stagnation pressure just after its injector plate, and that the turbine exit stagnation pressure is the same as the ambient static pressure. You can also assume that all the _gases_ are thermally and calorically perfect. If you need properties of the unburned reactants (liquid and gaseous) or of the combustion products (from either of the combustion chambers), use values **only** from the table below.

	ρ (kg/m³)	**c_p (J/kg·K)**	***MW***	γ
Liquid O_2	1170	1670	32.00	
Liquid CH_4	440.	3500	16.04	
Gaseous O_2			32.00	1.32
Gaseous CH_4			16.04	1.27
Gas Generator Products			14.53	1.19
Main Combustor Products			20.50	1.21

a) Determine the required mass flow rates for the oxidizer and fuel entering the main combustion chamber.

b) Determine the required mass flowrates for the oxidizer and fuel entering the gas generator, and shaft power produced by the turbine.

c) Determine the actual specific impulse (in seconds) of the rocket engine – based on all the propellant mass being consumed to operate the engine.

2. Solid Rocket Motor

A solid rocket motor (SRM) is being designed as a booster for a launch vehicle. At sea-level conditions (i.e., liftoff), the SRM must deliver 445 kN (~100 klb_f); in addition, the SRM must produce thrust for 2 minutes and 23 seconds. It has already been decided to use a neutral burning grain design, operating with a motor pressure of 74.9 bar and with a nozzle having an (exit-to-throat) area ratio of 13.9.

The SRM will use a composite <u>propellant</u> with the following <u>properties</u>:

ρ_{solid} (kg/m³)	$MW_{products}$	T_o (K)	$\gamma_{products}$
1750	20.1	2930	1.22

The regression rate of the propellant follows the standard (St. Robert's Law) expression, $r=ap^n$, with n=0.441 and a =0.157 mm s^{-1} when pressure is given in kPa.

You may assume the internal nozzle flow is quasi-1D, adiabatic and reversible.

a) Find the minimum propellant mass that will need to be loaded into the motor to meet these requirements.

b) Determine the throat diameter AND burning surface area required to meet the above requirements.

c) At lift-off, determine the thrust-to-weight (T/W) ratio of the SRM, assuming the mass of the structural elements of the SRM is 5.5% of the propellant mass.

d) Find the thrust that the SRM will produce just before it reaches its burnout altitude, where the local ambient pressure is 101 Pa.

Extra Credit. Heat Transfer

You have been asked to check the design of a regenerative cooling system for a TCA like that of the methane/oxygen rocket from problem 1 (note: *you don't need to use any results from problem 1 to solve this problem*).

The engine designers are planning to build the TCA with a 0.55 mm thick wall between the hot combustion products and the liquid methane coolant flow. The walls are to be composed of a Cu-based alloy, with a maximum safe operating temperature of 770 K (on the hot side of the wall). The properties of this alloy are given in the table below, along with the properties of an alternate alloy, stronger but less conducting.

Alloy	ρ (kg/m^3)	c_p (kJ/kgK)	k (W/mK)
Cu-based	8980	0.381	365
Alternate	8200	0.435	91.3

The designers want to check the material temperatures at a location near the nozzle throat. In this region, the engine designers have provided the following estimates for the hot combustion product flow through the TCA:

T (K)	ρ (kg/m^3)	c_p (kJ/kgK)	k (W/mK)	μ (kg/m/s)	u (m/s)	Diameter (m)
3200	2.9	2.4	0.43	1.1×10^{-4}	1100	0.22

and the following properties for the liquid methane flow through the coolant channels:

T (K)	ρ (kg/m^3)	c_p (kJ/kgK)	k (W/mK)	μ (kg/m/s)	u (m/s)	Hydraulic D (mm)
110	440	3.4	0.19	1.3×10^{-4}	22	1.9

In addition, your team has identified the following heat transfer correlation values as being reasonable for the conditions under consideration (with $Nu = C\, Re_D^m Pr^n$):

Flow	C	M	N
Hot	0.0214	0.80	0.39
Cold	0.0230	0.80	0.33

with D defined as the TCA diameter for the hot flow and the hydraulic diameter of the coolant channels for the cold flow.

Determine if the wall temperature on the hot-gas side will exceed the estimated safety limit of 770 K. Also provide estimates of the wall temperature on the coolant side of the combustor and the heat flux through the wall. Finally, if the designers choose to use the strong alloy, find the required TCA wall thickness if they wanted to produce the same wall temperatures as those achieved with the Cu-based alloy.

You may assume the TCA geometry can be reasonably modeled using a one-dimensional analysis approach.