Ideal Gas Mixtures

- Most propulsion applications do not involve one "pure" gas – involve gas mixtures
  - for example: air (O_2, N_2,..), engine exhaust (products of combustion)
- How to calculate properties of ideal gas mixture?
  - can use mixture averaged properties, e.g.,

$$p = \rho R_{mix} T \qquad R_{mix} = \frac{\overline{R}}{MW_{mix}}; \quad \boxed{MW_{mix} = \sum_i \chi_i MW_i} \quad \textbf{(II.13)}$$

mole fraction $\chi_i = n_i / n_{mix}$

  - or sum up

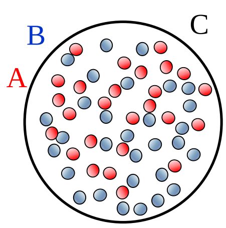
partial pressure

$$p = \sum_i p_i \qquad p_i = \rho_i R_i T$$
$$p_i = \chi_i p$$

overall gas is ideal if each individual gas is ideal

Calculating Properties of Mixtures

- What about other properties?
  - internal energy *E* or enthalpy *H*
  - e.g., gas C composed of part A and B



mixture averaged properties

$$\Delta H_{C_{1,2}} = m_{mix} \int_{T_1}^{T_2} c_{p_{mix}} dT$$

mass fraction

$$c_{p\,mix} = \sum_i Y_i c_{p_i} \qquad Y_i = m_i / m_{mix}$$

or

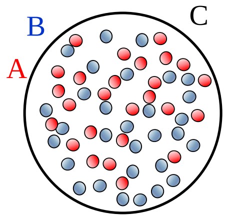
simple summation

$$\Delta H_{C_{1,2}} = \Delta H_{A_{1,2}} + \Delta H_{B_{1,2}}$$

$$\Delta H_{C_{1,2}} = m_A \{h_A(T_2) - h_A(T_1)\} + m_B \{h_B(T_2) - h_B(T_1)\}$$

$$\Delta H_{C_{1,2}} = m_A \int_{T_1}^{T_2} c_{p_A} dT + m_B \int_{T_1}^{T_2} c_{p_B} dT$$

Ideal Gas Mixture Entropy



- What about entropy S ?
  - use mixture averaged properties

$$\Delta S_{C_{1,2}} = m_{mix}\left[\int_{T_1}^{T_2} c_{p_{mix}} \frac{dT}{T} - R_{mix} \ln\left(\frac{p_2}{p_1}\right)\right]$$

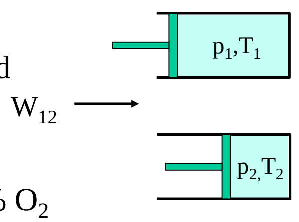
  - or sum up components

$$S_C(T,p) = S_A(T,?) + S_B(T,?)$$

all components have same T, but what pressure should we use for each component? **… their partial pressure**

$$\Delta S_{C_{1,2}} = m_A\left\{\int_{T_1}^{T_2} c_{p_A} \frac{dT}{T} - R_A \ln\left(\frac{p_{2A}}{p_{1A}}\right)\right\} + m_B\left\{\int_{T_1}^{T_2} c_{p_B} \frac{dT}{T} - R_B \ln\left(\frac{p_{2B}}{p_{1B}}\right)\right\}$$

Example



- **Given:** air at 1 atm and 300 K compressed to 10 atm and 700 K
- **Find:** change in entropy per unit mass
- **Assume:** synthetic air is 79% N_2 and 21% O_2 (by mole); N_2, O_2 are TPG and CPG under these conditions
- **Analysis:**

1) mix. avg properties

$$MW_{air} = 0.79(28\,kg/kmol) + 0.21(32\,kg/kmol)$$
$$= 28.85\,kg/kmol$$

$$R_{air} = \frac{\overline{R}}{MW_{air}} = \frac{8314\,J/kmolK}{28.8\,kg/kmol} = 288\,J/kgK$$

$$c_{p_{air}} \cong \frac{7}{2}R_{air} = 1.01\,kJ/kgK$$

low T diatomic gas

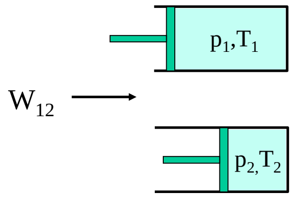
$$\Delta s_{12} = c_{p_{air}} \ln\left(\frac{T_2}{T_1}\right) - R_{air} \ln\left(\frac{p_2}{p_1}\right)$$

$$= 1.01\frac{kJ}{kgK}\ln\left(\frac{7}{3}\right) - 0.288\frac{kJ}{kgK}\ln\left(\frac{10}{1}\right)$$

$$\boxed{= 0.191\frac{kJ}{kgK}}$$

S increased…why?

Example



- **Analysis:**

  2) summation

$$\Delta S_{12,air} = \Delta S_{12,N_2} + \Delta S_{12,O_2}$$

$$m_{air}\Delta s_{12,air} = m_{N_2}\Delta s_{12,N_2} + m_{O_2}\Delta s_{12,O_2}$$

$$\Delta s_{12,air} = Y_{N_2}\Delta s_{12,N_2} + Y_{O_2}\Delta s_{12,O_2}$$

$$= 0.767\frac{kg_{N_2}}{kg_{air}}\left(0.1968\frac{kJ}{kg_{N_2}K}\right)$$

$$+0.233\frac{kg_{O_2}}{kg_{air}}\left(0.1722\frac{kJ}{kg_{O_2}K}\right)$$

$$\boxed{= 0.191\frac{kJ}{kgK}} \checkmark$$

$$Y_{N_2} = \chi_{N_2}\frac{MW_{N_2}}{MW_{air}} = 0.79\frac{28.01}{28.8} = 0.767$$

$$Y_{O_2} = 1 - Y_{N_2} = 0.233$$

$$\Delta s_{12,N_2} = \frac{8314}{28.01}\frac{J}{kg_{N_2}K}\left\{\frac{7}{2}\ln\left(\frac{7}{3}\right) - \ln\left(\frac{7.9}{0.79}\right)\right\}$$

$$= 0.1968\, kJ/kg_{N_2}K$$

$$\Delta s_{12,O_2} = \frac{8314}{32.0}\frac{J}{kg_{O_2}K}\left\{\frac{7}{2}\ln\left(\frac{7}{3}\right) - \ln\left(\frac{2.1}{0.21}\right)\right\}$$

$$= 0.1722\, kJ/kg_{O_2}K$$