IV. Rocket Propulsion Systems

D. Chemical Rocket Cycle Analysis

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Ideal Rocket Cycle Analysis

- Just like with air-breathing engines, we are interested in predicting rocket performance (thrust, impulse,…)
- Start by considering simplified, ***ideal*** rocket cycle with
  1) combustion chamber; and
  2) nozzle
- Idealizing assumptions
  1. steady flow, thermally and calorically perfect gas, constant properties (MW, γ)
  2. chemical reaction equivalent to constant pressure heating (reversible)
  3. nozzle expansion is 1-d, reversible and adiabatic (isentropic)

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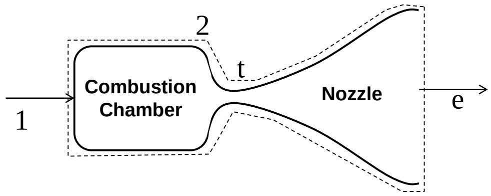
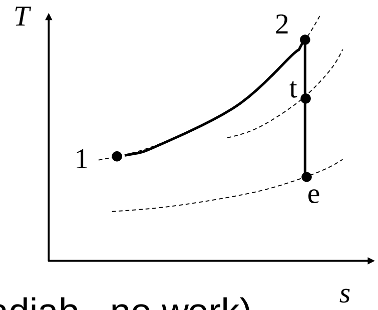
Ideal Rocket Cycle





- *Combustion Chamber*
  - energy conservation (adiab., no work)

$$\dot{m}\Delta h_R = \dot{m}\left(h_{o2} - h_{o1}\right) = \dot{m}c_p\left(T_{o2} - T_{o1}\right)$$

$$\boxed{T_{o2} = T_{o1} + \Delta h_R / c_p} \quad \textbf{(IV.9)}$$

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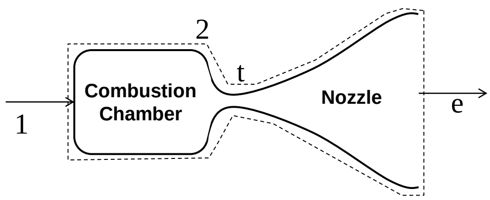
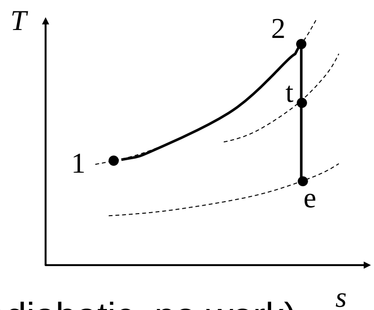
Ideal Rocket Cycle





- *Nozzle*
  - energy conservation (adiabatic, no work)

$$\dot{m}h_{o2} = \dot{m}\left(h_e + u_e^2/2\right) \;\Rightarrow u_e = \sqrt{2\left(h_{o2} - h_e\right)} = \sqrt{2c_pT_{o2}\left(1 - T_e/T_{o2}\right)}$$

  - reversible

$$\boxed{u_e = \sqrt{\frac{2\gamma}{\gamma - 1} RT_{o2}\left[1 - \left(\frac{p_e}{p_{o2}}\right)^{\gamma-1/\gamma}\right]}} \quad \textbf{(IV.10)}$$

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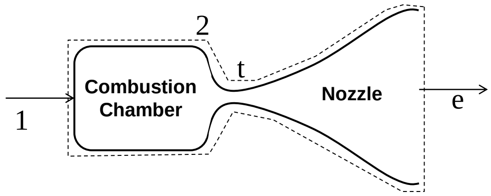
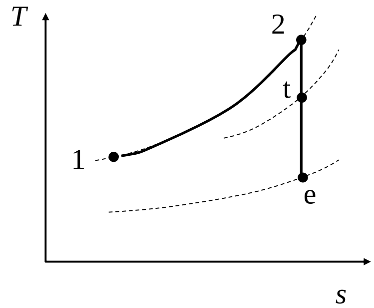
Ideal Rocket Cycle





- *Nozzle*
  - choked throat (M_t=1)

$$\boxed{\dot{m} = \rho u A = \frac{p_{o2}}{\sqrt{\left(\bar{R}/MW\right)T_{o2}}} A_t \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} \quad \textbf{(IV.11)}$$

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Maximizing Specific Impulse and Thrust

- Already saw $I_{sp} \sim u_{eq} \sim u_e$
- To get high u_e

$$u_e = \sqrt{\frac{2\gamma}{\gamma-1}\frac{\bar{R}}{MW}T_{o2}\left[1-\left(\frac{p_e}{p_{o2}}\right)^{\gamma-1/\gamma}\right]} \quad \text{(IV.10)}$$

1. low MW propellant
2. high p_o/p_e (high C.C. pressure, and large nozzle area ratio)

$$T_{o2} = T_{o1} + \Delta h_R / c_p \quad \text{(IV.9)}$$

3. high T_o (high $\Delta h_R / c_p$, chemical energy)

- Thrust ~ $\dot{m}u_e$; to get high $\dot{m}$

$$\dot{m} = \frac{p_{o2}}{\sqrt{\bar{R}T_{o2}/MW}} A_t \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \quad \text{(IV.11)}$$

1. large throat area, A_t
2. high chamber p_o
3. low combustion T_o
4. high propellant MW

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