



Thermodynamics Properties

- **Property**
 - any characteristic of a system which can be *quantitatively evaluated* and which is *related to energy* of system
 - examples: $m, V, E, p, T, S, H, \dots$
- **Independent Properties**
 - Question: How many *intensive* properties does it take to define a unique *state* for a *known* substance?
 - Answer: **Two** - for simple compressible substances
 - useful work done only by compression/expansion; no E&M fields, no liquid surface tension,...
- So if you know two properties, can predict the rest



State Equations

- Relate TD properties
 - e.g., $Y=Y(T,p)$
- **Examples**
 - Gibbs equation (from 1st and 2nd Laws)

$$ds = \frac{de}{T} + \frac{p}{T} dv \quad (\text{II.5})$$
 - caloric equations of state

$$h = h(T, \rho)$$

$$e = e(T, \rho)$$
- **Simplify by restricting ourselves to perfect gases**
 - obey **Perfect Gas** relation

$$\begin{aligned} p &= \rho RT \\ \text{or} \\ pv &= RT \end{aligned} \quad (\text{II.6})$$



Perfect (Ideal) Gases

- Thermal (virial) state equation

$$p = \rho RT = \rho \frac{\bar{R}}{M} T; \bar{R} = 8.3143 \text{ J/mol} \cdot \text{K} = 8.3143 \text{ kJ/kmol} \cdot \text{K}$$

$$= 1.9858 \text{ cal/mol} \cdot \text{K} = 1545.3 \text{ ft} \cdot \text{lb}_f / \text{lbmol} \cdot \text{K}$$

- “Energy” state equations (**Specific Heats**)

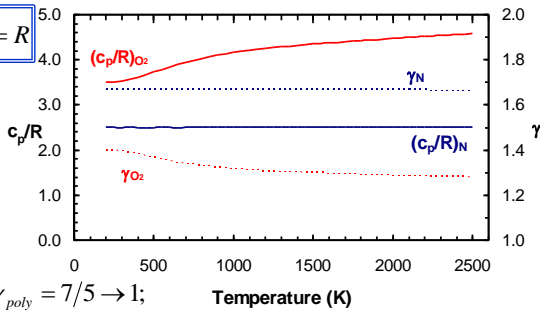
(II.7)

$$c_v \equiv \frac{de}{dT}; c_p \equiv \frac{dh}{dT}; c_p - c_v = R$$

$$\gamma \equiv \frac{c_p}{c_v} = \frac{c_p}{c_p - R} = \frac{1}{1 - R/c_p}$$

$$\Rightarrow \frac{c_p}{R} \left(\frac{\bar{c}_p}{\bar{R}} \right) = \frac{\gamma}{\gamma - 1}$$

bar means per mole



$$\gamma_{atom} = 5/3; \gamma_{diatom} = 7/5 \rightarrow 9/7; \gamma_{poly} = 7/5 \rightarrow 1;$$

Temperature (K)

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Perfect Gas – Entropic State Eq'n.

- Gibbs Eq. $Tds = de + pdv = de + \underbrace{pdv + (vdp - vdp)}_{d(pv) - vdp}$
 $= de + d(pv) - vdp = dh - vdp$

$$ds = \frac{dh}{T} - \frac{v}{T} dp$$

$$= \frac{c_p(T)dT}{T} - \frac{R}{p} dp$$

for a P.G.
 $dh = c_p dT$
 $pv = RT$

From state 1 to state 2

$$\int_{s_1}^{s_2} ds = s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)dT}{T} - R \int_{p_1}^{p_2} \frac{dp}{p} = \int_{T_1}^{T_2} \frac{c_p(T)dT}{T} - R \ln \frac{p_2}{p_1}$$

$$s_2 - s_1 = [\underbrace{\phi(T_2)}_{\text{fn of T only}} - \underbrace{\phi(T_1)}_{\text{fn of T only}}] - R \ln(p_2/p_1)$$

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fn of T only

fn of p only

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p-T-s State Equation

$$s_2 - s_1 = \Delta s_{12} = [\Delta \phi_{12}] - R \ln(p_2/p_1) = \int_{T_1}^{T_2} \frac{c_p(T)dT}{T} - R \ln(p_2/p_1)$$

Cal. Perf. $\Delta s_{12} = c_p \ln(T_2/T_1) - R \ln(p_2/p_1)$ (II.9)

• **Pressure ratio**

General

$$\frac{p_2}{p_1} = e^{(\Delta \phi_{12} - \Delta s_{12})/R}$$

Calorically Perfect

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{c_p/R} e^{-\Delta s_{12}/R}$$

– **if isentropic**
($\Delta s_{12}=0$)

$$\frac{p_2}{p_1} = e^{\Delta \phi_{12}/R}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{c_p/R} = \frac{\gamma}{\gamma-1}$$

(II.10)

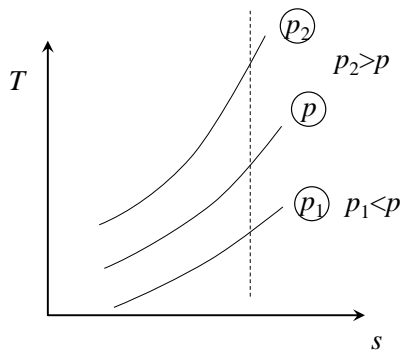
must be absolute p,T



State Diagrams

- Useful to be able to visualize/graph state relationships

– in engine (cycle) analysis, T-s an important diagram



$$Tds = c_p dT - vdp$$

$$dT = (T/c_p)ds + (v/c_p)dp$$

$$= \left. \frac{\partial T}{\partial s} \right|_p ds + \left. \frac{\partial T}{\partial p} \right|_s dp$$

>0
↑ w/T ↑

>0

s ↑ for T ↑
@constant p

T ↑ for p ↑
@constant s

(p)

(s)