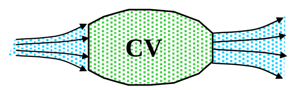
Thermodynamic Systems

- **Systems**
  - a **finite portion of matter** ***or*** **restricted portion of space** that we wish to examine
- **Control Mass** (Lagrangian Approach)
  - fixed portion of *matter* surrounded by real or imaginary boundaries
  - *matter can* ***not*** *cross boundary*

gas in a piston

- **Control Volume** (Eulerian Approach)
  - volume of *space* surrounded by real or imaginary boundaries
  - *matter* ***can*** *cross boundary*



flow through an engine

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Conservation Equations

- Govern mechanics and thermodynamics of systems
- Control Mass Laws
  - mass: can not create or destroy mass (e.g., neglect nuclear reactions)
  - momentum: Newton's Law, $F=ma$
  - energy: 1st Law of thermodynamics, $dE=\delta Q-\delta W$
  - entropy: 2nd Law, $dS=\delta Q/T+\delta P_s$
- Propulsion systems generally employ fluid flow
  - need to write conservation laws in terms for **Control Volumes**

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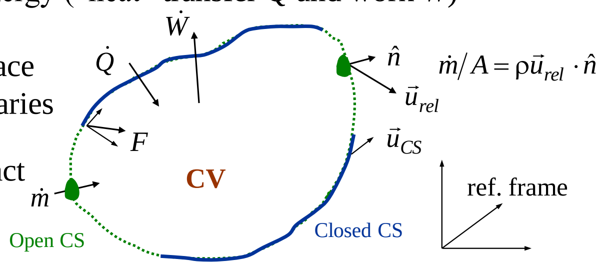
Reynolds Transport Theorem - RTT

- Provides general form for converting **conservation laws** from control mass to **control volumes**
- Take arbitrary control volume (**CV**); can be moving
  - mass and energy ("heat" transfer Q and work W) can cross control surface (**CS**) boundaries
  - forces also act on CS and mass in CV



$\vec{u}_{rel}$ = velocity of material crossing CS relative to motion of CS

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RTT Equation

- Take any **extensive property** **B**, that follows a "conservation" law and its **intensive version** **β** (per mass); can show

$$\left.\frac{dB}{dt}\right|_{CM} = \frac{d}{dt}\int_{CV}\rho\beta\, dV + \int_{CS}\rho\beta\left(\vec{u}_{rel}\cdot\hat{n}\right)dA \qquad \textbf{(II.1)}$$

Replace with appropriate Control Mass Conservation Law

Storage term (rate of increase inside CV)

Net flux of property leaving CV, carried by flow (outflow - inflow)

- Always leads to **PICO** relationship

Production + **I**nput = **C**hange (in time) + **O**utput

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Mass Conservation

- If property of interest is mass

$$B = m,\quad \beta = \frac{dB}{dm} = \frac{d(m)}{dm} = 1$$

- From **RTT**

$$\left.\frac{dB}{dt}\right|_{CM} = \frac{d}{dt}\int_{CV} \rho\beta\, dV + \int_{CS} \rho\beta\left(\vec{u}_{rel}\cdot\hat{n}\right) dA$$

$$\left.\frac{d(m)}{dt}\right|_{CM} = \frac{d}{dt}\int_{CV} \rho 1\, dV + \int_{CS} \rho 1\left(\vec{u}_{rel}\cdot\hat{n}\right) dA$$

- **CM:** $\left.\frac{d(m)}{dt}\right|_{CM} = 0$

Integral Control Volume Form of Mass Conservation

- **CV:**

$$0 = \frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\left(\vec{u}_{rel}\cdot\hat{n}\right) dA \qquad \textbf{(II.2)}$$

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Simplified Mass Conservation

$$0 = \frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho\left(\vec{u}_{rel}\cdot\hat{n}\right) dA$$

- **Uniform flow (at CS)** - no variations across flow

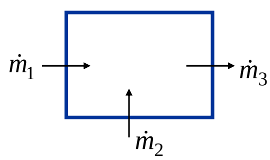
$$\int_{CS(t)} \rho\left(\vec{u}_{rel}\cdot\hat{n}\right) dA = \sum_{outlets} \rho u_{rel} A - \sum_{inlets} \rho u_{rel} A$$

$$= \sum_{outlets} \dot{m} - \sum_{inlets} \dot{m}$$

- Add **Steady-State**

$$\frac{d}{dt}\int_{CV} \rho\, dV = 0$$

$$\Rightarrow \sum_{outlets} \dot{m} = \sum_{inlets} \dot{m}$$



$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

~~P~~I=~~C~~O $\rightarrow$ **I**nput = **O**utput

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Conservation of Momentum

- **Linear momentum**

$$\vec{B} = m\vec{u}, \quad \vec{\beta} = \vec{u}$$

- **RTT** then gives

$$\left.\frac{d(m\vec{u})}{dt}\right|_{CM} = \frac{d}{dt}\int_{CV} \rho\vec{u}\,dV + \int_{CS} \rho\vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA$$

- Use **Newton's Law**

$$\left.\frac{d(m\vec{u})}{dt}\right|_{CM} = \sum \vec{F}$$

total force acting *on fluid*

$$\sum \vec{F}_{CV} = \frac{d}{dt}\int_{CV} \rho\vec{u}\,dV + \int_{CS} \rho\vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA$$

momentum transferred *to fluid*

Input = Change + Out - In ~~P~~I=CO (0)

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Force Terms

- Examine different forces that can act on matter in our control volume

e.g., gravity; e.g., pressure, shear,...

$$\sum \vec{F}_{onCV} = \sum_{onCV} \vec{F}_{body} + \sum_{onCS} \vec{F}_{surface}$$

- **Body forces**

$$\vec{F}_{body} = \int_{CV} \rho\vec{f}\,dV \quad \text{with } \vec{f} = \text{body force/mass,}$$

i.e, **acceleration**

- **Surface forces**
  - **free surfaces**: not connected to solid body crossing control surface
  - **connected surfaces**: solid boundaries where there are reaction forces

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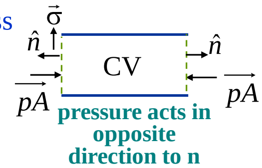
Surface Forces

- **Free surfaces**

normal stress, shear stress

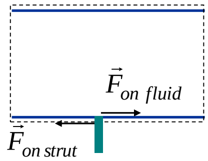
$$\vec{F}_{surface} = -\int_{CS} p\hat{n}dA + \int_{CS} \vec{\sigma}_{shear} dA$$

0 if inviscid



- **Connected solid surfaces**
  - force on fluid is reaction force (inverse) of force on solid body

$$\vec{F}_{on\ fluid} = -\vec{F}_{on\ solid\ body}$$



- Combine

Integral Control Volume Form of Momentum Conservation

$$\vec{F}_{\substack{solid\ body \\ on\ fluid}} - \int_{CS} p\hat{n}dA + \int_{CS} \vec{\sigma}_{shear} dA + \int_{CV} \rho\vec{f}dV = \frac{\mathrm{d}}{\mathrm{d}t}\int_{CV} \rho\vec{u}dV + \int_{CS} \rho\vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA \quad \text{(II.3)}$$

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Conservation of Energy

- **Energy:** microscopic + macroscopic forms of energy

internal energy

$$\mathrm{B} = E_o = E + E_{kinetic} = E + \frac{1}{2}mu^2$$

energy per mass→ $\beta = e_o = e + \frac{u^2}{2}$

- **RTT** then gives

$$\left.\frac{\mathrm{d}\left(E_{tot}\right)}{\mathrm{d}t}\right|_{CM} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{CV} \rho e_{tot} dV + \int_{CS} \rho e_{tot}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA$$

- Use 1st Law Thermodynamics for energy conservation of control mass

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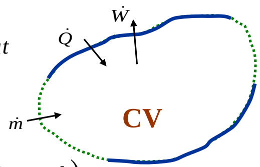
1st Law of Thermodynamics

- Differential form $dE_{CM} = \delta Q_{in} - \delta W_{out}$

$$\left.\frac{dE}{dt}\right|_{CM} = \frac{\delta Q_{in}}{dt} - \frac{\delta W_{out}}{dt} = \dot{Q}_{in} - \dot{W}_{out}$$



- Into RTT

$$\dot{Q}_{in} - \dot{W}_{out} = \frac{d}{dt}\int_{CV} \rho e_o dV + \int_{CS} \rho e_o \left(\vec{u}_{rel} \cdot \hat{n}\right) dA$$

- But work is related to forces (F·x) acting on CV
  - already examined some kinds of forces
  - let's relate them to work terms

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Work and Forces

- Relationship

$$\dot{W} = \vec{F} \cdot \frac{d\vec{x}}{dt} = \vec{F} \cdot \vec{u}$$

Since we defined work as positive when done **BY** fluid

- **Body forces**

$$\dot{W}_{body} = - \int_{CV} \rho \vec{f} \cdot \vec{u} dV$$

- **Fluid forces** (stresses)

$$\dot{W}_{press} = \int_{CS} p\left(\vec{u} \cdot \hat{n}\right) dA$$

"Flow Work"

$$\dot{W}_{shear} = -\int_{CS} \vec{\sigma} \cdot \vec{u} dA$$

- **Reaction Forces**
  - lump into "useful" work term, e.g., shaft work

$$\dot{W}_{shaft}$$

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Energy Conservation

- Combine into RTT result (*neglect shear forces*)

$$\dot{Q}_{in} - \dot{W}_{shaft} + \int_{CV} \rho\vec{f}\cdot\vec{u}dV - \int_{CS} p(\vec{u}\cdot\hat{n})dA = \frac{d}{dt}\int_{CV} \rho e_o dV + \int_{CS} \rho e_o(\vec{u}_{rel}\cdot\hat{n})dA$$

Similar Form

Stagnation Enthalpy

Combine flow work and energy flux

$$\rho e_o + \rho\frac{p}{\rho} = \rho\left(e_o + \frac{p}{\rho}\right) = \rho h_o \qquad h_o = h + \frac{u^2}{2}$$

$$\dot{Q}_{in} - \dot{W}_{shaft} + \int_{CV} \rho\vec{f}\cdot\vec{u}dV - \int_{CS} p(\vec{u}-\vec{u}_{rel})\cdot\hat{n}dA = \frac{d}{dt}\int_{CV} \rho e_o dV + \int_{CS} \rho h_o(\vec{u}_{rel}\cdot\hat{n})dA$$

- For reference frame moving with control volume at *constant velocity and no body forces*

$$\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt}\int_{CV} \rho e_o dV + \int_{CS} \rho h_o(\vec{u}\cdot\hat{n})dA \qquad \textbf{(II.4)}$$

~~P~~^0 I=CO — **I**n - **O**ut — **C**hange — **O**ut - **I**n, of energy in mass

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