Rocket Propulsion

Thrust Coefficient, Characteristic Velocity and Ideal Nozzle Expansion

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Thrust Coefficient

- Define thrust coeff.

$$\boxed{c_\tau \equiv \frac{\tau}{A_t p_o}} \quad \textbf{(IV.12)}$$

- Also steady thrust

$$\tau = \dot{m}u_e + \left(p_e - p_a\right)A_e$$

- Can combine with **ideal nozzle** results (IV.10,11)

$$u_e = \sqrt{\frac{2\gamma}{\gamma-1}RT_o\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]}$$

$$\dot{m}_{choked} = A_t\frac{p_o}{\sqrt{RT_o}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Ideal Thrust Coefficient

$$\boxed{c_{\tau,ideal} = \gamma\sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{{}^{\gamma+1}\!/_{\gamma-1}}\left[1-\left(\frac{p_e}{p_o}\right)^{{}^{\gamma-1}\!/_{\gamma}}\right]}+\left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}}$$

(IV.13)

- Ideal thrust coefficient is only function of
  - – γ, ε ($=A_e/A_t$), p_a/p_o
  - – recall p_e/p_o = fn(ε)
- Note: $c_\tau \neq$ fn(T_o, MW)

a nozzle characteristic

- Thrust coeff. depends mostly on pressure distribution in thrust chamber
  - – from normalizing thrust by p_oA_t

Characteristic Velocity

- Can define similar parameter to characterize combustor
  - – **characteristic velocity**

$$\boxed{c^* \equiv \frac{p_oA_t}{\dot m}}$$

(IV.14)

- Can also write **ideal characteristic velocity**
  - – using (IV.11)

$$\dot m_{choked} = A_t\frac{p_o}{\sqrt{RT_o}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\boxed{c^*_{ideal} = \sqrt{\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}\frac{\overline{R}}{MW}T_o}}$$

(IV.15)

- Note: c^* = fn(T_o, MW, γ) **a propellant combustion property**
- Also $c_\tau c^* = \frac{\tau}{p_oA_t}\frac{p_oA_t}{\dot m} = \frac{\tau}{\dot m} \Rightarrow \boxed{c_\tau c^* = u_{eq}}$ **(IV.16)**

Liquid Bipropellants - Examples

Oxidizer	BP/FP (°C)	Fuel	BP/FP (°C)	Combustor Temperature (K)	Bulk Avg. Density (g/cm^3)	C* (m/s)	Isp (s)	u_{eq} (m/s)
O_2	-183/-218	H_2	-253/-259	3010	0.3	2420	390	3830
O_2		RP-1	~210/-50	3680	1.0	1810	300	2940
O_2		UDMH	63/-58	3600	1.0	1860	310	
O_2		NH_3	-33/-78	3080	0.9	1800	295	
F_2	-188/-220	H_2		3960	0.5	2560	410*	4020
F_2		Hydrazine	113/1.4	4680	1.3	2210	363*	
N_2O_4	21/-12	MMH	86/-53	3390	1.2	1750	288*	
N_2O_4		RP-1		3450	1.3	1650	275	2700

Optimum performance; 1000psia (6.94MPa) combustor; $p_e=p_a=14.7$ psia (1 atm)

UDMH=Unsymmetrical dimethyl hydrazine $(CH_3)_2NNH_2$ **Hydrazine**=N_2H_4

MMH=Monomethyl hydrazine $CH_3NH\text{-}NH_2$ **NH_3**=Ammonia

***Hypergolic Mixture** (ignites on contact)

- generally $c^* < u_{eq}$
  - because with well designed system $c_\tau > 1$

Ideal Thrust Coefficient – Mom. vs. Press.

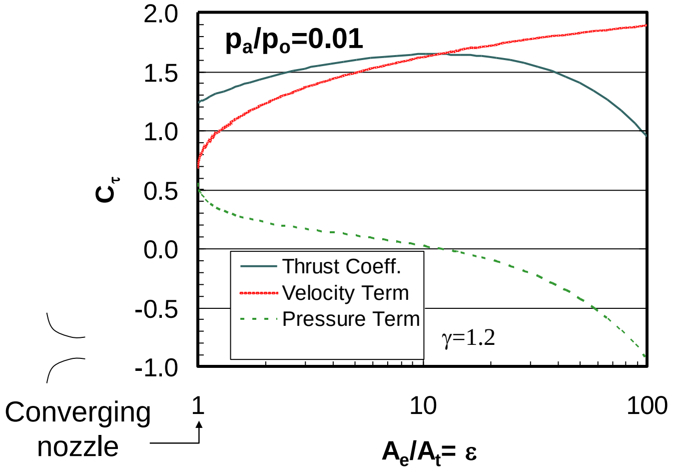
- Re-examine (IV.13)

$$c_{\tau,ideal} = \underbrace{\sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_o}\right)^{\frac{\gamma-1}{\gamma}}\right]}}_{\dot{m}u_e/p_oA_t} + \underbrace{\left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}}_{(p_e-p_a)A_e/p_oA_t}$$

- 1st term = contribution to thrust by exit velocity/momentum
- 2nd term = contribution to thrust by exit pressure

Comparison of Terms

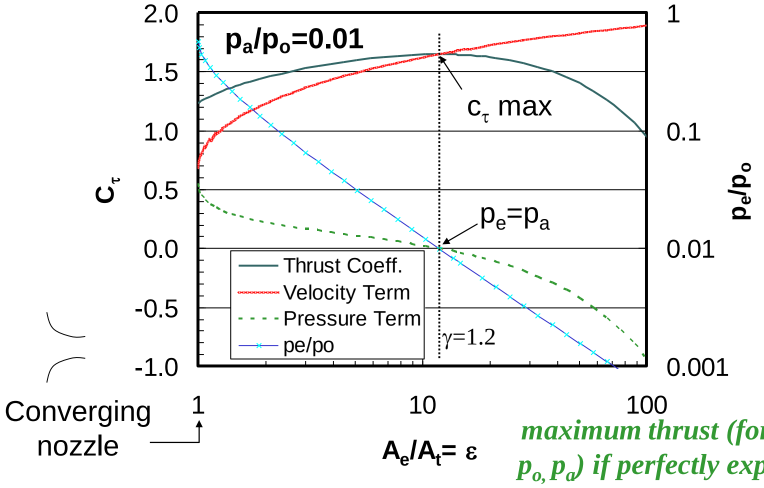
- Compare terms for different nozzle designs



- Velocity term always provides thrust (+)
- Pressure term can increase or decrease thrust

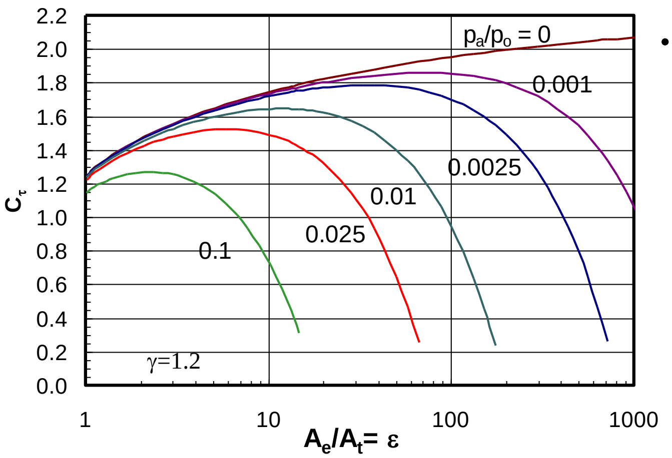
Comparison of Terms

- Look at exit versus ambient pressure



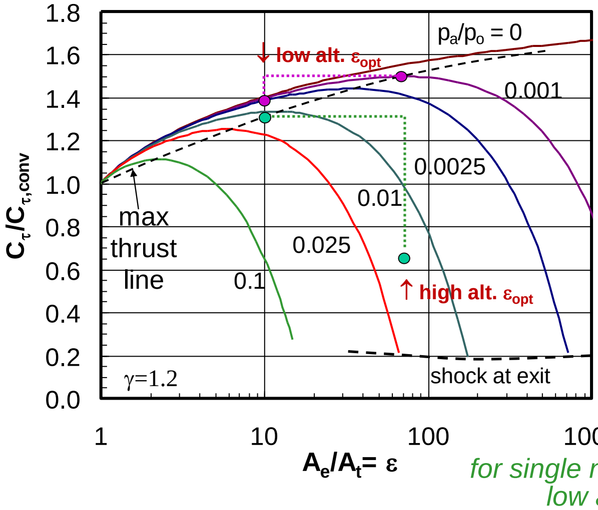
maximum thrust (for fixed p_o, p_a) if perfectly expanded

Effect of Ambient Pressure on c_τ



- Can get higher thrust coefficient by:
  - reducing ambient pressure
  - increasing rocket pressure

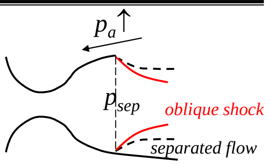
Normalize by Converging Nozzle c_τ

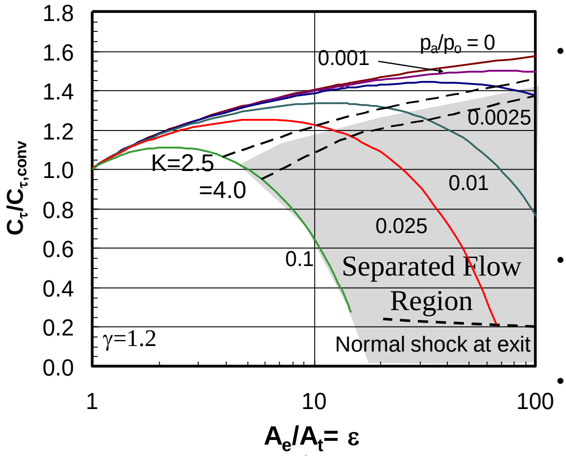


- Large ε needed for optimum c_τ for small p_a/p_o
- ε for optimum c_τ (or I_{sp}) varies with altitude (p_a)
  - for p_o=1000 psia
  - $p_a/p_o \cong$
    - **0.015** **sea-level**
    - **0.001** **60,000 ft**
  - performance at multiple altitudes?

for single nozzle, best ε is closer to low alt. optimum value

Flow Separation





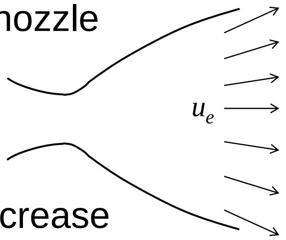
- For p in nozzle enough below p_a, flow (b.l.) separates
  - occurs in over-expanded operation and before normal shock would enter
  - expansion essentially ends at separation (lower ε)
- Summerfield* found oblique shock enters nozzle for $\boxed{K \equiv p_a/p_{e,sep} = 2.5\text{–}4}$ **(IV.17)**

  $\Rightarrow \quad p_e/p_o < 25\text{–}40\% \;\; p_a/p_o$
- Kalt and Bendall** another empirical criteria (one of many) **(IV.18)** $\boxed{p_{sep}/p_a = \tfrac{2}{3}\left(p_o/p_a\right)^{-0.2}}$

*Summerfield et al., *Jet propulsion* **24** (1954)

***J. Spacecraft and Rockets* **2** (1965)

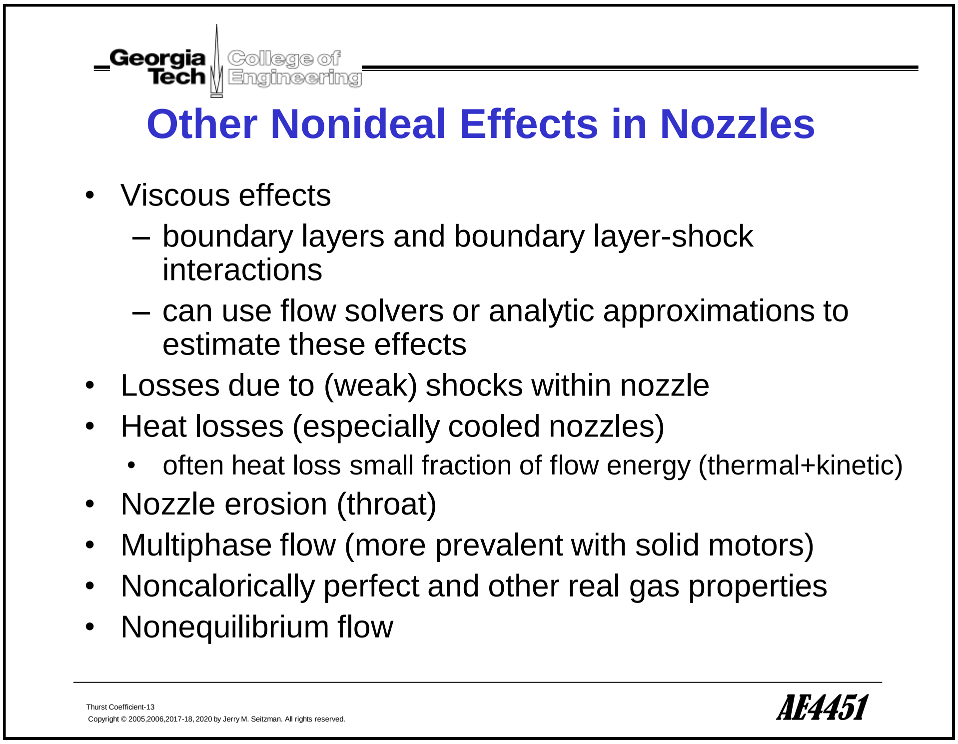
Flow Divergence



- Generally flow leaving a rocket nozzle is not directed in axial direction
  - would require excessive length nozzle
- Thus some of the momentum increase produced by the nozzle is not aligned with nozzle axis ⇒ **thrust reduction/loss**
- For uniform $|u_e|$ can apply correction factor λ

$$\boxed{\tau = \lambda \dot{m} u_e + \left(p_e - p_a\right) A_e} \quad \textbf{(IV.19)}$$

will reduce c_τ



Other Nonideal Effects in Nozzles

- Viscous effects
  - – boundary layers and boundary layer-shock interactions
  - – can use flow solvers or analytic approximations to estimate these effects
- Losses due to (weak) shocks within nozzle
- Heat losses (especially cooled nozzles)
  - often heat loss small fraction of flow energy (thermal+kinetic)
- Nozzle erosion (throat)
- Multiphase flow (more prevalent with solid motors)
- Noncalorically perfect and other real gas properties
- Nonequilibrium flow