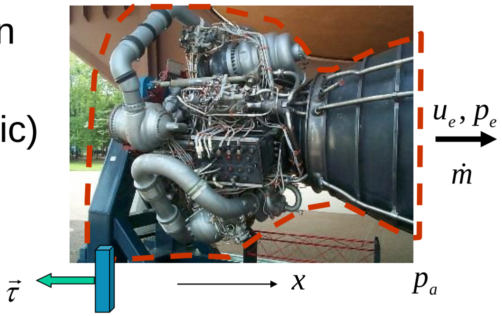
IV. Rocket Propulsion Systems

B. Thrust and Impulse

Static Thrust

- Consider engine on ground test stand
  - not moving (static)
  - steady
  - quasi 1-d flow



momentum conservation

$$\vec{F}_{\substack{solid\,body\\ on\ fluid}} - \int_{open} p\hat{n}dA + \int_{open} \vec{\sigma}_{shear}dA + \int_{CV} \rho\vec{f}dV = \frac{\mathrm{d}}{\mathrm{d}t}\int_{CV} \rho\vec{u}dV + \int_{CS} \rho\vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA$$

$$-\left(-\tau\right)-\left(p_e - p_a\right)A_e + 0 + 0 = \frac{\mathrm{d}}{\mathrm{d}t}\int_{CV} \rho 0 dV + \dot{m}u_e$$

$$\boxed{\tau = \dot{m}u_e + \left(p_e - p_a\right)A_e} \quad \textbf{(IV.1)}$$

Equivalent Exhaust Velocity

- Definition

$$\boxed{u_{eq} \equiv \frac{\tau}{\dot{m}} \overbrace{= u_e + \frac{(p_e - p_a)A_e}{\dot{m}}}^{\textit{for steady, uniform/1-d}}} \quad \textbf{(IV.2)}$$

  - combines momentum change and pressure force terms
  - written for convenience

$$\Rightarrow \tau = \dot{m}u_{eq}$$

Impulse

- Using the definition of **Impulse**

$$I \equiv \int_{t_{start}}^{t_{end}} F dt$$

  - total momentum imparted by a force acting over time
  - using thrust force, and substituting equiv. velocity

$$\boxed{I = \int_{t_{start}}^{t_{end}} \tau \, dt = \int_{t_{start}}^{t_{end}} \dot{m}u_{eq} \, dt} \quad \textbf{(IV.3)}$$

  - assuming *steady* nozzle and ambient conditions

$$I = u_{eq}\int_{t_{start}}^{t_{end}} \dot{m}dt = \underset{\text{total mass of expelled propellant}}{m_p} u_{eq}$$

Specific Impulse

- Definition

$$\boxed{I_{sp} \equiv I/m_p = u_{eq}}$$ (IV.4a)

for steady

units of velocity

  - divided by propellant mass to determine rocket performance per kg of propellant it has to carry
  - higher I_{sp} means less propellant required
    - thus more payload that can be carried
    - or lighter, smaller rocket can be used
- Normalization
  - to get same value in all unit systems, typically normalize I_{sp} by Earth's gravitational constant (*gravity at Earth's surface*)

$$g_e \approx 9.81 m/s^2$$

not *g* at rocket location

$$\boxed{I_{sp} = \left(\frac{I}{m_p}\right)\Big/ g_e = u_{eq}/g_e}$$ (IV.4b)

for steady

units of time (seconds)

Liquid Bipropellants - Examples

Oxidizer	BP/FP (°C)	Fuel	BP/FP (°C)	Combustor Temperature (K)	Bulk Avg. Density (g/cm^3)	C* (m/s)	Isp (s)	u_{eq} =? (m/s)
O_2	-183/-218	H_2	-253/-259	3010	0.3	2420	390	3830
O_2		RP-1	~210/-50	3680	1.0	1810	300	2940
O_2		UDMH	63/-58	3600	1.0	1860	310	
O_2		NH_3	-33/-78	3080	0.9	1800	295	
F_2	-188/-220	H_2		3960	0.5	2560	410*	4020
F_2		Hydrazine	113/1.4	4680	1.3	2210	363*	
N_2O_4	21/-12	MMH	86/-53	3390	1.2	1750	288*	
N_2O_4		RP-1		3450	1.3	1650	275	2700

Optimum performance; 1000psia (6.94MPa) combustor; p_e=p_a=14.7 psia (1 atm)

UDMH=Unsymmetrical dimethyl hydrazine $(CH_3)_2NNH_2$ **Hydrazine**=N_2H_4

MMH=Monomethyl hydrazine $CH_3NH\text{-}NH_2$ **NH_3**=Ammonia

*__Hypergolic Mixture__ (ignites on contact)

Solid Propellants - Examples

Propellant	Metal (wt %)	Combustion Temperature (K)	Density (g/cm^3)	Isp (s)	u_{eq} (m/s)
Double Base		2530	1.6	230	2260
DB/AP	Al (20)	3870	1.8	265	
Polyurethane-AP	Al (20)	3480	1.8	265	
PBAN-AP	Al (16)	3480	1.8	263	
HTPB-AP		3000	1.8	250	2450
HTPB-AP	Al (17)	3480	1.9	265	2600

Double Base=homogeneous mixture nitroglycerine-nitrocellulose
$C_3H_5(NO_2)_3$-$C_6H_7O_2(NO_2)_3$
AP=Ammonium Perchlorate **PBAN**=Polybutadiene-Acrylic Acid-Acrylonitrile Terpolymer
HTPB=Hydroxy-terminated Polybutadiene

Specific Impulse: Ranges

- Chemical rockets
  - liquid bipropellants typically higher I_{sp} than solids
  - typically 200-400 seconds at sea level exhaust
  - increase by ~17% for vacuum exhaust (40:1 nozzle expansion area ratio)
    - $\Rightarrow I_{sp,max} \leq$ 480 seconds
    - $\Rightarrow u_{eq,max} \leq$ 4700 m/s (15,400 ft/s)
- **Electrical rocket systems** can have
  - $I_{sp} >$ 1000-3000 seconds
  - $u_{eq,max} \geq$ 10,000-30,000 m/s (33,000-98,000 ft/s)
  - but thrust limited by available power source