Compressors

Cascade Analysis and Velocity Triangles

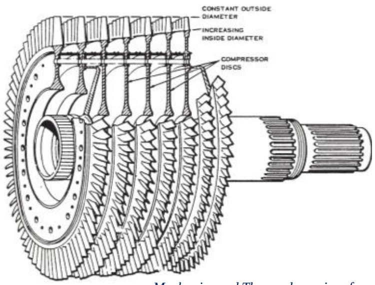
Compressor Analysis

- Need to determine change in azimuthal (swirl) velocity to (from V.5)

$$\dot{W}/\dot{m} = \Delta(uc_\theta)_{1,2}$$

- At the pitchline, and assuming a *constant* pitchline radius ($r_1 \approx r_2$), $u_m \equiv U$

$$\boxed{\dot{W}/\dot{m} = U\Delta c_{\theta 1,2}} \quad \textbf{(V.8)}$$

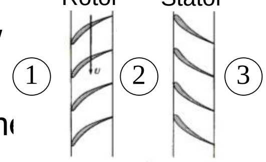


Mechanics and Thermodynamics of Propulsion, Hill and Peterson

- Find Δc_θ by analyzing a blade row in the 2-d cascade flowfield

Cascade Analysis

- You have previously analyzed flow over a “blade” (an airfoil/wing!!)
  – but in the blade’s reference frame
- But here there are moving (e.g. rotor) and stationary (stator) blades
  – e.g., compressor stage shown
  $1 \rightarrow 2$ rotor
  $2 \rightarrow 3$ stator
- Use flow angles and velocity triangles to visualize changes between ref. frames



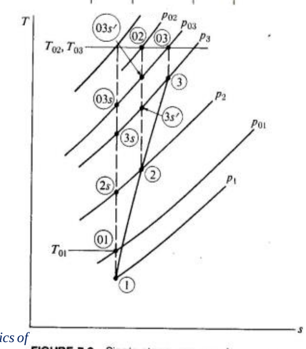


FIGURE 7.9 Single-stage compression process.

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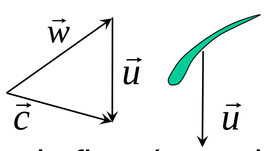
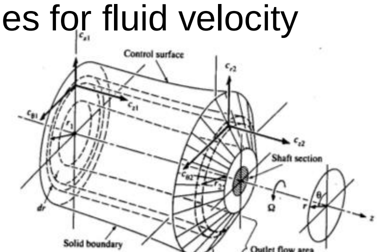
Compressor (Cascade) Flow Angles

- Recall, two reference frames for fluid velocity
  – engine’s $\vec{c}$
  – blade’s $\vec{w}$
  – with

$$\vec{c} = \vec{w} + \vec{u}$$

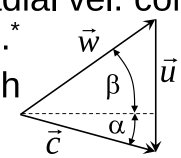


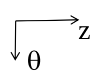


“abs” swirl downward can still have swirl upward relative to rotor

- For cascade flow (no radial vel. component)
  – use u to define +θ dir.*
  – define angles for each ref. frame (α and β)

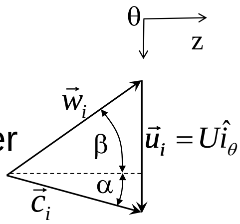
as shown here, $\beta < 0$





*sometimes (e.g., in UK) α >0 in U direction, β >0 in opposite direction

Velocity Triangles



- Rotor motion in θ direction, so ref. frame change has no effect on other directions $\Rightarrow$ $w_{zi} = c_{zi}$ **(V.9)**
- Rotor vel. constant in cascade flow (like fixed r), and let $|u_i| = U$

$\vec{c}_i = \vec{w}_i + \vec{u}$

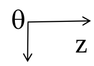
as shown here, $w_{\theta i} < 0$

$\Rightarrow$ $c_{\theta i} - w_{\theta i} = U$ **(V.10)**

- Also have general trigonometric relations
  – e.g.,

$$c_{\theta i} = c_i \sin\alpha_i = c_{zi} \tan\alpha_i$$
$$w_{\theta i} = w_i \sin\beta_i = w_{zi} \tan\beta_i$$
$$w_{zi} = w_i \cos\beta_i = c_{zi} = c_i \cos\alpha_i$$
(V.11)

Velocity Triangle Analysis



- Apply (V.9-11) to both blade rows of compressor stage
- 1: rotor inlet

  (11) $\Rightarrow c_{\theta_1} = c_{z_1} \tan\alpha_1$
- 2: between rotor and stator

  (9,11) $\Rightarrow w_{\theta_2} = c_{z_2} \tan\beta_2$

  (10) $\Rightarrow c_{\theta_2} = U + w_{\theta_2}$

  $c_{\theta_2} = U + c_{z_2} \tan\beta_2$ **(V.12)**

  with (11) $\Rightarrow$ $\tan\alpha_2 = \dfrac{U}{c_{z_2}} + \tan\beta_2$ **(V.13)**
- 3: stator outlet

  (11) $\Rightarrow c_{\theta_3} = c_{z_3} \tan\alpha_3$

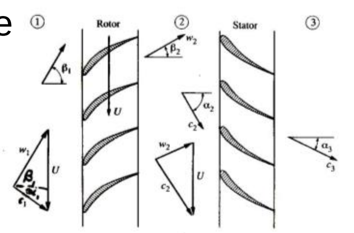


FIGURE 7.7 Mean radius section of a compressor stage. The absolute exit angle fror

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$w_{zi} = c_{zi}$ (9)

$c_{\theta i} - w_{\theta i} = U$ (10)

$c_{\theta i} = c_{zi} \tan\alpha_i$
$c_{\theta i} = c_i \sin\alpha_i = c_{zi} \tan\alpha_i$ (11)
$w_{\theta i} = w_i \sin\beta_i = w_{zi} \tan\beta_i$

Compressors

Stage Characteristics

Single-Stage Characteristics (Axial Compressor)

- Goal - how is compressor performance, e.g., Pr_c, affected by changes in operating conditions
- Start by analyzing single-stage of compressor
- **Rotor** (1→2)
  - Euler $\dot{W}_R = \dot{m}U\left(c_{\theta_2} - c_{\theta_1}\right) = \dot{m}\left(h_{o2} - h_{o1}\right)$
    $U\Delta c_{\theta_{1,2}} = \Delta h_{o_{1,2}}$
- **Stator** (2→3) $\dot{W}_S = 0 \Rightarrow h_{o3} = h_{o2}$
- So for stage $\boxed{\Delta h_{o_{1,3}} = \Delta h_{o_{1,2}} = U\Delta c_{\theta_{1,2}}}$ **(V.14)**
- From vel.triangle analysis (e.g., V.12) $\Delta c_{\theta_{1,2}} = U + c_{z_2}\tan\beta_2 - c_{z_1}\tan\alpha_1$
  if constant axial velocity $= U + c_z\left(\tan\beta_2 - \tan\alpha_1\right)$

Compressor Stage Pressure Ratio

- Recall pressure ratio for adiabatic compressor with thermally and calorically perfect gas

$$\frac{p_{o3}}{p_{01}} = \left[1+\eta_{st}\left(\frac{T_{o3}}{T_{o1}}-1\right)\right]^{\gamma/\gamma-1} = \left[1+\eta_{st}\left(\frac{T_{o3}-T_{o1}}{T_{o1}}\right)\right]^{\gamma/\gamma-1}$$

$$= \left[1+\eta_{st}\frac{\Delta h_{o_{1,3}}}{c_p T_{o1}}\right]^{\gamma/\gamma-1}$$

$$c_p = \frac{\gamma R}{(\gamma-1)} \qquad = \left[1+\eta_{st}\frac{(\gamma-1)U^2}{\gamma R T_{o1}}\frac{\Delta h_{o_{1,3}}}{U^2}\right]^{\gamma/\gamma-1}$$

$$\frac{\Delta h_{o_{1,3}}}{U^2} = \frac{\Delta c_{\theta_{1,2}}}{U} \qquad Pr_{stage} \equiv \boxed{\frac{p_{o3}}{p_{o1}} = \left[1+\eta_{st}(\gamma-1)\frac{U^2}{\gamma R T_{o1}}\frac{\Delta c_{\theta_{1,2}}}{U}\right]^{\gamma/\gamma-1}} \quad \textbf{(V.15)}$$

Compressor Stage Pressure Ratio

$$Pr_{stage} = \left[1+\eta_{st}(\gamma-1)\frac{U^2}{\gamma R T_{o1}}\underbrace{\frac{\Delta c_{\theta_{1,2}}}{U}}_{\equiv\psi}\right]^{\gamma/\gamma-1}$$

- Comp. stage pressure ratio depends on
  1. **stage loading coefficient**, ψ
     - Δc_θ and U
  2. blade Mach number, $U/\sqrt{\gamma R T_{o1}}$
     - rotor blade speed ($U = r\Omega$)
     - stage inlet stagnation temperature, T_{o1}
  3. stage efficiency, η_{st}

Stage Loading and Flow Coefficients

- **Stage Loading Coeff.**

$$\psi \equiv \frac{\Delta h_{o_{1,3}}}{U^2} = \frac{\Delta c_{\theta_{1,2}}}{U} \quad \textbf{(V.16)}$$

  – interpretation

$$\frac{\text{Work/mass}}{\text{KE rotor}} \sim \frac{\text{swirl increase}}{\text{rotor velocity}}$$

  – typical axial compressor designs ~ 0.3-0.5

- **Flow Coeff.**
  – already showed for const. axial vel., fixed r_m

$$\psi = \frac{\Delta c_{\theta_{1,2}}}{U} = 1 + \frac{c_z}{U}\left(\tan\beta_2 - \tan\alpha_1\right)$$

$$\frac{c_z}{U} \equiv \phi \quad \textbf{(V.17)}$$

$$\psi = 1 + \phi\left(\tan\beta_2 - \tan\alpha_1\right) \quad \textbf{(V.18)}$$

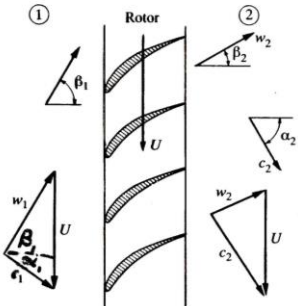
  – typical axial compressor designs ~ 0.4-0.8

Blade Design Influence

- What controls stage loading? $\psi = \frac{\Delta c_{\theta_{1,2}}}{U} = 1 + \phi\left(\tan\beta_2 - \tan\alpha_1\right)$

1. $\phi = c_z/U$ $\quad \downarrow \Rightarrow \psi\uparrow$
   - relative flowrate through stage, proportional to **mass flow rate,** vs blade speed (r, Ω)
2. α_1 $\quad \downarrow \Rightarrow \psi\uparrow$
   - **inlet flow angle** - from previous stage or inlet guide vanes (IGVs)
3. β_2 $\quad$ *less neg.* $= \downarrow \Rightarrow \psi\uparrow$
   - flow angle leaving rotor blade in rotor reference frame depends on **blade design angle**

increases rotor turn angle



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Compressor Example

- Axial air compressor **stage** with design point created for:
  1. 8,000. rpm rotor with 0.30 m mean design radius
  2. 100. m/s axial velocity (constant through stage), 36.4 m/s inlet tangential velocity
  3. 50.° rotor blade trailing edge angle (wrt to axis and opposite to rotation), and 20.° stator blade's trailing edge angle
  4. 95% estimated adiabatic stage efficiency @ design point
  5. 298 K inlet temperature
- Determine at design point:
  1. rotor speed (m/s)
  2. stage flow coefficient
  3. Stage loading coeff. and required power per flowrate (kJ/kg)
  4. stagnation pressure ratio across stage
- Assume - calorically/thermally perfect gas, zero radial velocity

Compressor Example

- Rotor speed

$$U = \Omega r = \left[N(rpm) \times \frac{2\pi\, rad}{rev.} \right] \times r$$

$$= 8{,}000 \frac{rev}{60\sec} 2\pi \frac{rad}{rev.} 0.3m = \boxed{251 m/s}$$

- Flow coefficient

from (V.17)

$$\phi = \frac{c_z}{U} = \frac{100.m/s}{251m/s} = \boxed{0.398}$$

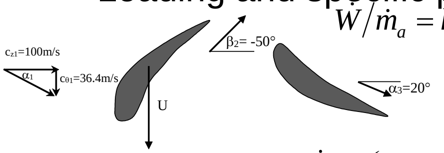
- Loading and specific power

$$W/\dot{m}_a = h_{o3} - h_{o1} = U^2\psi \quad \text{from (V.16)}$$

$$\psi = 1 + \phi(\tan\beta_2 - \tan\alpha_1) \quad \text{from (V.18)}$$

$$\psi = 1 + 0.398(\tan^{-} 50° - 0.364) = \boxed{0.381}$$

-1.19



$$\tan\alpha_1 = \frac{36.4}{100} \Rightarrow \alpha_1 = 20°$$

$$\frac{\dot{W}}{\dot{m}_a} = \left(251\frac{m}{s}\right)^2 0.381 = 24{,}065 \frac{m^2}{s^2}\frac{kg}{kg} = \boxed{24.1\frac{kJ}{kg}}$$

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Compressor Example

- Pressure Ratio

from (V.15) $$Pr_{st} = \left[1+\eta_{st}(\gamma-1)\frac{U^2}{\gamma R T_{o1}}\frac{\Delta c_{\theta_{1,2}}}{U}\right]^{\gamma/\gamma-1}$$

Blade M = 0.73

$$= \left[1+0.95(1.4-1)\left(\frac{251 m/s}{\sqrt{1.4(288\, J/kgK)298K}}\right)^2 0.381\right]^{3.5}$$

$$Pr_{st} = \boxed{1.3}$$

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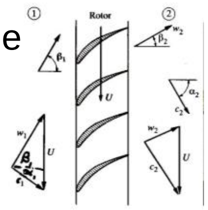
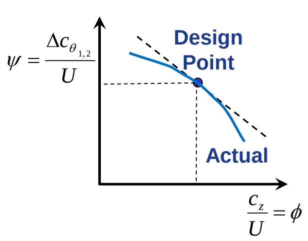
Single Stage Compressor: Off-Design

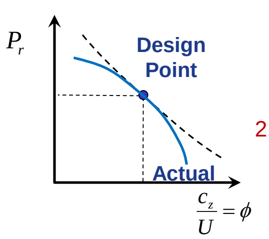
- Near design point, flow ~follows blade

$$\frac{\Delta c_{\theta_{1,2}}}{U} = 1+\frac{c_z}{U}(\tan\beta_2 - \tan\alpha_1) = 1 - C_a \times \frac{c_z}{U}$$

$$\frac{p_{o3}}{p_{o1}} = \left[1+\eta_{st} C_b U^2\left(1-C_a\frac{c_z}{U}\right)\right]^{\gamma/\gamma-1}$$







1. As flowrate changes @ fixed U, work doesn't
   - e.g. for higher mass flowrate ⇒ less work done per unit mass,so less pressure rise
2. As operation moves farther from design point, flow deviates more from blade
   - efficiency drops, drastically as stall grows

Single-Stage (Axial) Compressor Map

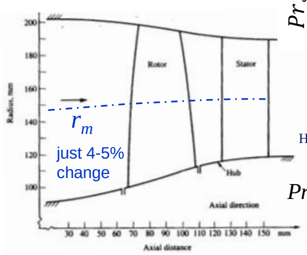
- Measured single-stage performance (for transonic stage)
  - $c_z \rightarrow$ **corrected mass flow rate**

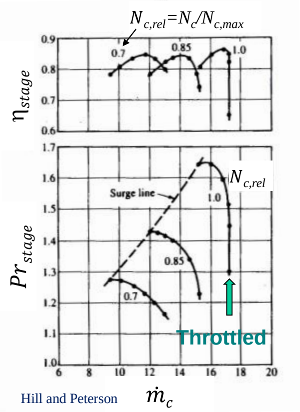
$$\dot{m}_c = \dot{m}\sqrt{\theta}/\delta \quad \textbf{(V.19)} \qquad \theta \equiv T_{o1}/T_{ref} \qquad \delta \equiv p_{o1}/p_{ref}$$

- $M_b \rightarrow$ **corrected RPM**

$$N_c = \left(N/\sqrt{\theta}\right) \quad \textbf{(V.20)}$$

- Note change in blade radii
- Drops in η
  - at high RPM and $\dot{m}$
    - throttled
  - other reasons?





$$Pr_{st} = \left[1 + \eta_{st}(\gamma - 1)\frac{U^2}{\gamma R T_{o1}}\frac{\Delta c_{\theta_{1,2}}}{U}\right]^{\gamma/\gamma-1}$$

Centrifugal (Single-Stage) Compressor Map

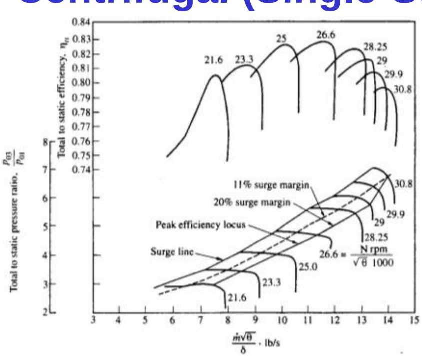


FIGURE 9.23 Performance of a centrifugal compressor. (Adapted from Kenny [14].)

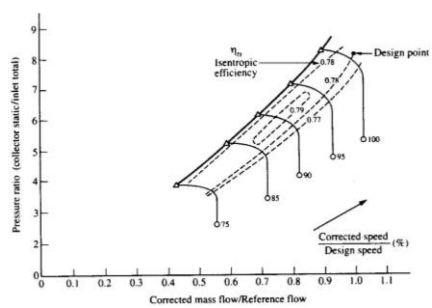


FIGURE 9.22 Performance of centrifugal compressor whose rotor is shown in Fig. 9.21.

Mechanics and Thermodynamics of Propulsion, Hill and Peterson

- Can apply similar approach to analyze characteristics of centrifugal/radial compressors
  - **much higher achievable stage pressure ratio**
  - lower peak efficiencies
  - Pr, η less sensitive to corrected mass flow rate (flatter)

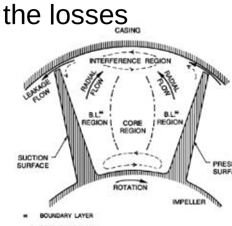
also less susceptible to FOD

Limitations of Cascade Analysis

- Analysis methods presented here only consider flow behavior at pitchline
- Does not account for radial variations (from root to tip of blade) in
  - flow properties (speed, flow angle, pressure,…)
  - blade speed
- Edge effects, flow separation,…

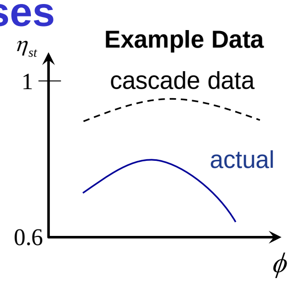
Efficiency Losses

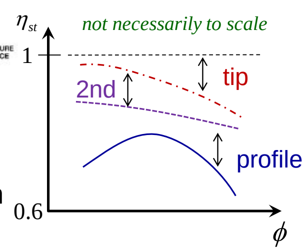
- **Profile (Blade) Losses (p_o)**
  - boundary layers
  - flow not following blade (separation)
  - shock waves in flow
  - but not explain all the losses
- **Tip Losses**
  - flow around blade tip (clearance between blade and casing or hub); like wing tip vortices
- **Secondary Flow**
  - 3-d flow, often related to boundary layers – especially at casing/hub, can also be induced by tip flow



Hydrodynamics of Pumps, Christopher Brennen 1994

Example Data

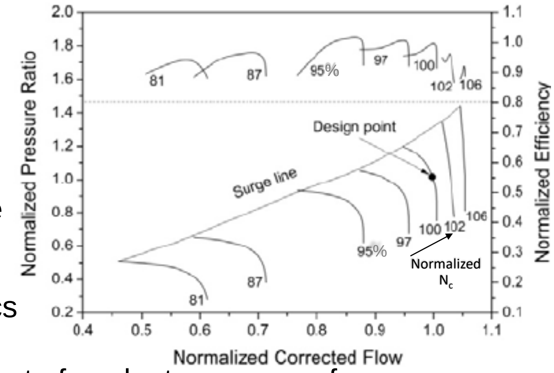




Compressors

Multistage Performance

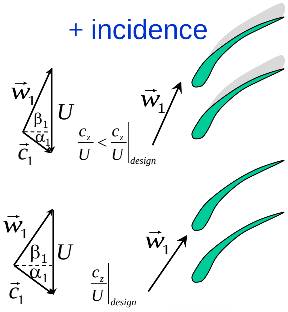
Multistage Compressors

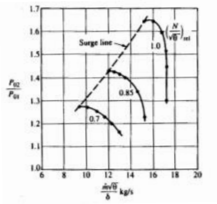


- Typical axial compressors have multiple stages on same spool
  - e.g., 3-8
- Overall performance of multistage compressor has similar characteristics as single-stage
  - BUT since the input of each stage comes from a previous stage, need to design them "together"
- Also as we go away from the design point interesting things can happen…

Multistage Compressors

- What can happen in a multistage (axial) compressor
  - exit of one stage influences following stage
- Consider: 1st stage with $\dot{m}_c < \dot{m}_{c,design}$ at design RPM
  $\Rightarrow Pr_{st} > Pr_{st_{design}}$
  - so 2nd stage sees lower $\dot{m}$ and even lower c_z
    - higher $p \Rightarrow \rho\uparrow$ and so $c_z \propto \dot{m}/\rho\downarrow$
  - pushes c_z even lower for 3rd stage…
  - gets worse with each successive stage until positive incidence stall

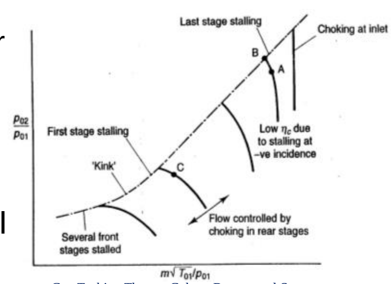




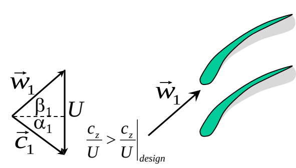
Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Multistage Compressors

- If $\dot{m} > \dot{m}_{design}$ for first stage, later stages will eventually have negative incidence stall
- Similar behavior if sudden change in RPM for fixed $\dot{m}$ *but less likely*
- If c_z/U increases too much, final stages throttled (choking) and can give pressure drop (negative pressure rise)
  - so low overall Pr_c
- Thus multistage compressors susceptible to significant off-design performance problems

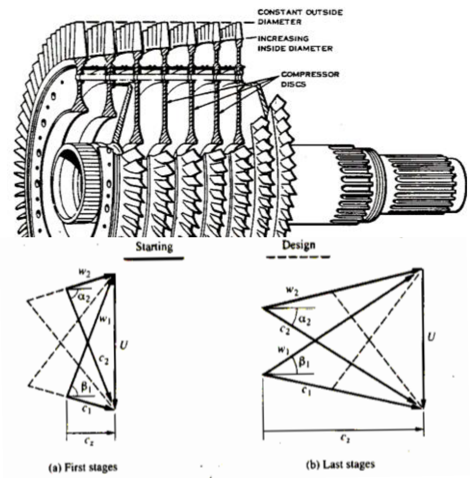


Gas Turbine Theory, Cohen, Rogers and Saravanamuttoo



Starting Issues

- What happens in multistage compressor at start up (low RPM)?
- Initially p and ρ ~constant but area increasing down-stream
- So low c_z in first stages, but high in last stages
  - so early stages positive incidence stall
  - last stages negative incidence and choking



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Solutions for Multistage Compressors

- Problems
  - off-design $\dot{m}$ for given RPM
  - incidence angle does not match blade angle
  - poor RPM for given c_z with many stages
- Solutions
  - **bleed valves** to remove mass between stages
  - **variable** stator and inlet guide **vane angles**
  - **LP and HP** sections with different RPM

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Compressors

Stability and Surge

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Compressor Stability (Surge)

- Dynamic instability problem
  - leads to periodic mass flow rate and pressure fluctuations

5. Surge is in evidence at takeoff. Flame duration is very short—milliseconds

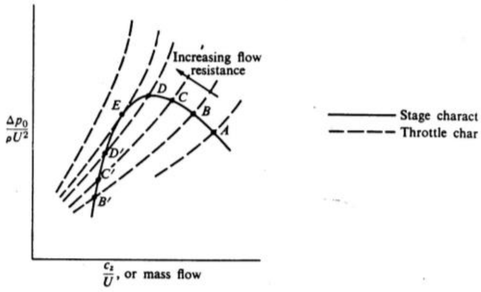
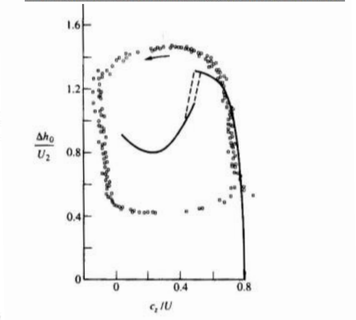


FIGURE 7.16 Pressure-rise characteristic compared with throttle characteristic of d resistance.

Mechanics and Thermodynamics of Propulsion, Hill and Peterson



Surge cycle showing reverse flow periods. (Courtesy Greitzer [5].)

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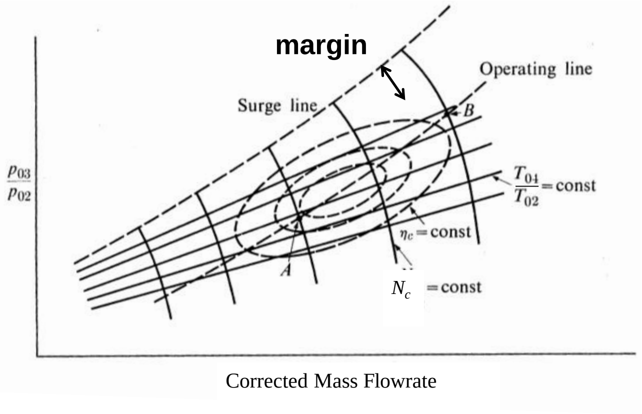
Surge Margin

- Leads to constraint on engine operation
- Need sufficient margin to avoid surge during engine operation



adapted from Mechanics and Thermodynamics of Propulsion, Hill and Peterson

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Compressor Transient Operation

- Surge more likely to happen during acceleration transient
  - limits power increase rate
- Blowout limits decel rate