Turbines

Overview

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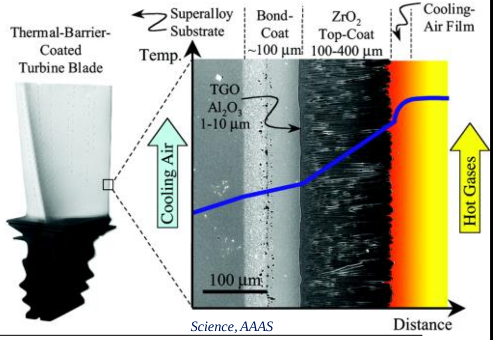
Turbine Overview

- Configurations (axial, radial, mixed), analysis and other issues similar to compressors
- Compared to compressors
  - higher loading $\Delta h_o/U^2$ (or specific work) and pressure ratio per stage - why?
    - favorable pressure gradient
  - usually much higher temperature inlet
    - higher temperature materials (strength) and/or blade cooling



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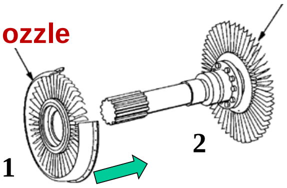
Turbine Analysis

- Similar to compressor analysis
  - Euler turbomachinery equations still hold, e.g., V.5,6

$$\dot{W} = \dot{m}\left[u_{i+1}c_{\theta_{i+1}} - u_i c_{\theta_i}\right]$$

$$\Delta\left(h_0\right)_{i,i+1} = \Delta\left(uc_\theta\right)_{i,i+1}$$



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

  - use cascade flow and pitch(mean) line approach for modeling Δc_θ
  - essentially same equations and approach as compressor, but new state numbering order
    - 1→2 stator=nozzle
    - 2→3 rotor

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Turbines

Cascade Analysis and Turbine Characteristics (Maps)

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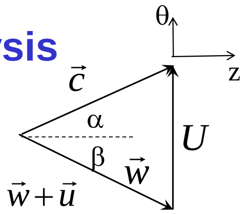
Turbine Cascade Analysis



- Let rotor move upward (flip sign convention); again fixed r, u_i=U

  $w_{z_i} = c_{z_i} \qquad c_{\theta_i} - w_{\theta_i} = U$ $\qquad \vec{c} = \vec{w} + \vec{u}$

- Therefore for **rotor**, and constant c_z

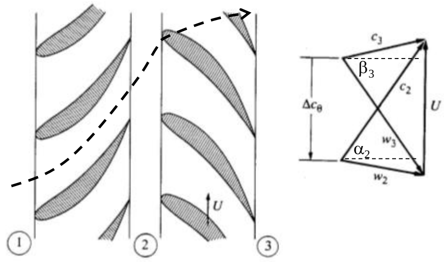
  $c_{\theta_2} = c_z \tan\alpha_2$

  (V.21) $c_{\theta_3} = U + c_z \tan\beta_3$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = 1 + \frac{\overset{\phi}{c_z}}{U}\left(\tan\beta_3 - \tan\alpha_2\right)$$

$$\psi \quad = \Delta h_{o_{1,3}} \big/ U^2$$

same form of blade loading eqn for turbine as compressor (1,2→2,3)



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Stage Pressure Ratio

- For adiabatic turbine with TPG/CPG

<0 for turbine

$$Pr_t = \frac{p_{o1}}{p_{03}} = \left[1 + \frac{1}{\eta_{st}}\left(\frac{T_{o3} - T_{o1}}{T_{o1}}\right)\right]^{\gamma/1-\gamma} = \left[1 + \frac{1}{\eta_{st}} \frac{(\gamma-1)U^2}{\gamma R T_{o1}} \frac{\Delta h_{o_{1,3}}}{U^2}\right]^{\gamma/1-\gamma}$$

>1 as written

$$\frac{\Delta h_{o_{1,3}}}{U^2} = \frac{\Delta c_{\theta_{2,3}}}{U}$$

$$\frac{p_{o1}}{p_{o3}} = \left[1 + \frac{\gamma - 1}{\eta_{st}} \underbrace{\frac{U^2}{\gamma R T_{o1}}}_{M_b^2} \underbrace{\frac{\Delta c_{\theta_{2,3}}}{U}}_{\psi}\right]^{\gamma/1-\gamma} \quad \text{(V.22)}$$

- Stage pressure ratio still depends on
  1. ψ= f(***U***= *r*$\boldsymbol{\Omega}$, $\boldsymbol{\Delta c_\theta}$)
  2. blade M=f($r\Omega$, $\boldsymbol{T_{o1}}$)
  3. η_{st}

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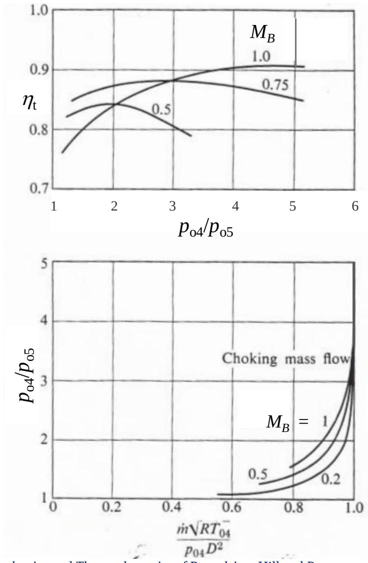
Axial Turbine Maps

- Important observations:

1. Larger stage pressure ratios (and possibly efficiencies) then compressors
2. Efficiency remains high for larger range of p ratios
3. For fixed RPM, larger pressure drop at higher mass flowrate
   - more work extracted per unit mass
4. Turbines (nozzle) can run choked (max. $\dot{m}_c$)



adapted from Mechanics and Thermodynamics of Propulsion, Hill and Peterson

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Turbine Example

- **Given**:
  - combustor exhaust with T_o=1800 K
  - 1st stage of turbine with c_z=447 m/s and nozzle producing 54° exit flow angle
  - rotor with blade speed of 526 m/s at pitchline (r_m=0.45 m) and flow exit angle of -49° in blade reference frame
- **Find:**
  1. Specific work produced by turbine stage and ψ
  2. Flow angle entering rotor in rotor ref frame
  3. T_o exit
  4. N (rpm)
- **Assume:**
  - axial velocity constant through stage
  - constant radius pitchline
  - combustion products TPG/CPG with MW 28.9, γ=1.33

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Turbine Example

- Specific work, ψ?

$$\dot{W}/\dot{m} = \Delta h_{o_{1,3}} = \Delta c_{\theta_{2,3}} U \left(= \psi U^2\right)$$

from (V.21) $\Delta c_{\theta_{2,3}} / U = 1 + \frac{c_z}{U}\left(\tan\beta_3 - \tan\alpha_2\right)$

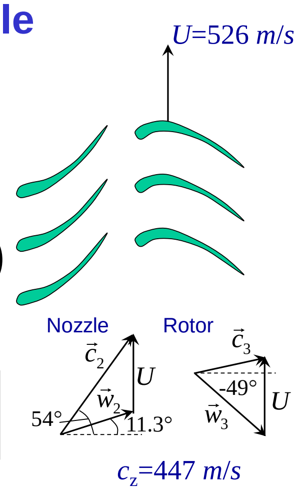
$$\psi = 1 + \frac{447}{526}\left(\tan^{-} 49^\circ - \tan 54^\circ\right)$$

ϕ=0.85

|ψ| and ϕ higher than for axial compressors

$$\boxed{\psi = -1.15}$$

$$\dot{W}/\dot{m} = -1.15\left(526\frac{m}{s}\right)^2 = \boxed{-317.4\frac{kJ}{kg}}$$



- β_2?

from (V.13) $\beta_2 = \tan^{-1}\left(\tan\alpha_2 - 1/\phi\right) = \boxed{11.3^\circ}$

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Turbine Example

- T_{o3} = ?

$$c_p = \frac{\gamma}{\gamma - 1} R = 1160 \frac{J}{kgK}$$

$$T_{o_3} = T_{o_1} + \Delta h_{o_{1,3}} / c_p$$

$$= 1800K - \left(317.4 kJ/kg\right) / \left(1.16 kJ/kgK\right) = \boxed{1530K}$$

- N?

$$N = \frac{U}{r_m}\frac{30}{\pi}$$

$$= \frac{526\, m/s}{0.45 m}\frac{30}{\pi} = \boxed{11{,}160 rpm}$$

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Turbines

Blade Design and Compressor-Turbine Matching

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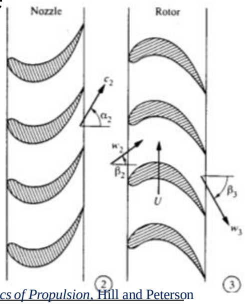
Blade Design

- We have TWO blade parameters to design $\psi = 1 + \phi(\tan\beta_3 - \tan\alpha_2)$
  1. rotor trailing edge angle ($\sim\beta_3$)
  2. nozzle trailing edge angle ($\sim\alpha_2$)
     – assume α_1 set by upstream device
     – β_2 can be found from α_2 and ϕ using geometric relations (like V.11) and ref. frame change, e.g.,

$$c_{\theta_2} = c_z \tan\alpha_2 \qquad w_{\theta_2} = c_{\theta_2} - U = c_z \tan\beta_2$$

$$c_{\theta_3} = c_z \tan\alpha_3 \qquad w_{\theta_3} = c_{\theta_3} - U = c_z \tan\beta_3$$



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

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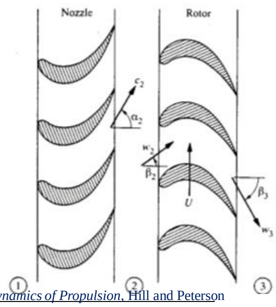
Blade Design: Degree of Reaction

- One choice for the two design variables:
  1. ψ $\qquad \psi = 1 + \phi(\tan\beta_3 - \tan\alpha_2)$
  2. $M_{2,rel}$ $\qquad M_{2,rel} = \dfrac{c_{z_2}}{a_2}\sqrt{1 + (\tan\alpha_2 - 1/\phi)^2}$

  – an approach we might have used for compressor stage
- Another choice for the two design variables are:
  1. Degree of reaction, ***R***
  2. Stage exit condition, α_3



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Degree of Reaction

- Recall $\qquad R \equiv \Delta h_{rotor} / \Delta h_{stage}$
  – allows us to distribute load (static pressure change) between rotor and nozzle (or stator)
  – how to relate static enthalpy change to azimuthal velocity changes?
    - **ΔKE** !! $\quad h_o = h + \mathrm{v}^2/2$

  – for stationary blade, no work done

$$\Delta h_o = 0 \Rightarrow \Delta h = -\Delta KE$$

- e.g., nozzle blade

if c_z constant and negligible c_r (so axial turbine only)

$$h_2 - h_1 = \left(\cancel{c_{z_1}^2} + c_{\theta_1}^2 + \cancel{c_{r_1}^2}\right)/2 - \left(\cancel{c_{z_2}^2} + c_{\theta_2}^2 + \cancel{c_{r_2}^2}\right)/2 = \left(c_{\theta_1}^2 - c_{\theta_2}^2\right)/2$$

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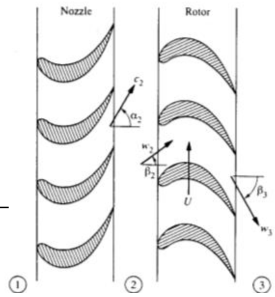
Degree of Reaction (Turbine)

$c_{z_3} = c_{z_2} = c_{z_1}$



- Rotor blades??
  - are "stationary" in rotor's reference frame

$$h_3 - h_2 = \left(w_{\theta_2}^2 - w_{\theta_3}^2\right)/2$$

- Reaction

$$R = \frac{h_3 - h_2}{h_3 - h_1} = \frac{h_3 - h_2}{\left(h_{o3} - c_3^2/2\right) + \left(h_{o1} - c_1^2/2\right)}$$

if $c_1 \approx c_3$ or $\Delta c^2/2 << \Delta h$

$$\cong \frac{h_3 - h_2}{h_{o3} - h_{o1}} = \frac{w_{\theta_2}^2 - w_{\theta_3}^2}{2U\left(c_{\theta_3} - c_{\theta_2}\right)}$$

$\left(c_{\theta_3} - c_{\theta_2}\right) = \Delta c_{\theta_{2,3}}$

(V.23)

$$R = \frac{w_{\theta_2}^2 - w_{\theta_3}^2}{2U^2\left[1 + \phi\left(\tan\beta_3 - \tan\alpha_2\right)\right]}$$

$\left[1 + \phi\left(\tan\beta_3 - \tan\alpha_2\right)\right] = \psi$

relates design blade angles to azimuthal KE change

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Impulse Turbine

$$R = \frac{\Delta h_{23}}{\Delta h_{13}} = \frac{w_{\theta_2}^2 - w_{\theta_3}^2}{2U^2\left[1 + \phi\left(\tan\beta_3 - \tan\alpha_2\right)\right]}$$

- $R = 0$
  - all the pressure change occurs across the nozzle, or the nozzle creates high KE

$$w_{\theta_2}^2 - w_{\theta_3}^2 = 0 \quad \Rightarrow w_{\theta_3} = -w_{\theta_2}$$

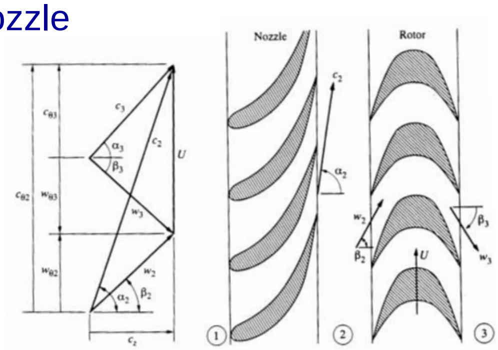
$$c_z \tan\beta_3 = -c_z \tan\beta_2$$

(V.24)

$$\Rightarrow \boxed{\beta_3 = -\beta_2 \qquad \Delta w_{\theta_{2,3}} = -2w_{\theta_2}}$$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = \frac{\Delta w_{\theta_{2,3}}}{U} = \frac{2\left(U - c_z \tan\alpha_2\right)}{U}$$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = 2\left(1 - \frac{c_z}{U}\tan\alpha_2\right)$$



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Impulse Turbine

- So for impulse turbine, blade loading coeff.

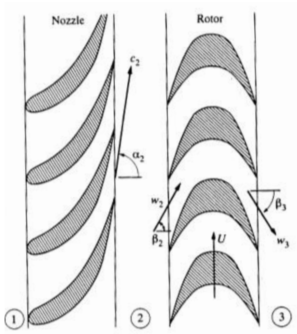
$$\frac{\Delta h_{o\,stage}}{U^2} = \frac{\Delta c_{\theta_{23}}}{U} = 2\left(1-\phi\tan\alpha_2\right) \quad \textbf{(V.25)}$$

- Relates blade loading to nozzle exit angle

$$\tan\alpha_2 = \frac{1}{\phi}\left(1-\frac{\psi}{2}\right) \qquad (\psi<0)$$

- From ϕ equation, rotor blade angles given by

$$-\tan\beta_3 = \tan\beta_2 = \tan\alpha_2 - \frac{1}{\phi} \quad \textbf{(V.26)}$$



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Impulse Turbine

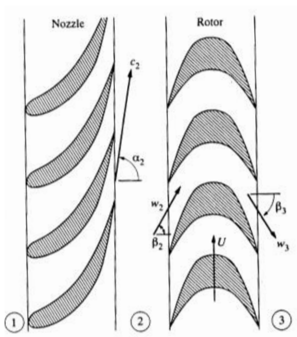
$$\frac{\Delta c_{\theta_{23}}}{U} = 2\left(1-\underbrace{\frac{c_z}{U}\tan\alpha_2}_{=c_{\theta_2}/U}\right)$$

- To get largest power per unit mass flow rate $\Rightarrow$ large α_2
  - tends to produce high velocities and p_o losses
  - practical limit, ~70°
- Further <u>possible</u> constraint
  - **no exit swirl** ($\Rightarrow c_{\theta 3}=0$)

$$\Delta c_{\theta_{2,3}} = c_{\theta_3} - c_{\theta_2} \;\Rightarrow\; \Delta c_{\theta_{2,3}} = -c_{\theta_2}$$

$$\Rightarrow \frac{\Delta c_{\theta_{23}}}{U} = 2\left(1+\frac{\Delta c_{\theta_{23}}}{U}\right) \Rightarrow \frac{\Delta h_{o_{stage}}}{U^2} = -2 \quad \textbf{(V.27)}$$

$$\Rightarrow \tan\alpha_2 = 2\left(U/c_z\right),\;\; \tan\beta_3 = U/c_z$$



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50% Reaction Turbine

- $R = 0.5$
  - balanced p drop across stage

$$R = \frac{w_{\theta_2}^2 - w_{\theta_3}^2}{2U\Delta c_{\theta_{2,3}}}$$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = 1 + \frac{c_z}{U}\left(\tan\beta_3 - \tan\alpha_2\right)$$

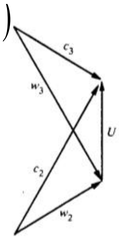
$$U\Delta c_{\theta_{2,3}} = w_{\theta_2}^2 - w_{\theta_3}^2$$

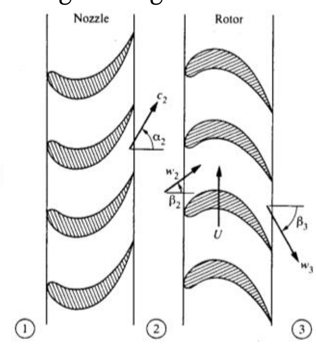
$$= \left(w_{\theta_2} + w_{\theta_3}\right)\left(w_{\theta_2} - w_{\theta_3}\right)$$

$$= \left(c_z \tan\alpha_2 - U + c_z \tan\beta_3\right)\left(-\Delta c_{\theta_{2,3}}\right)$$

$$\Rightarrow \tan\beta_3 = -\tan\alpha_2 \qquad \textbf{(V.28)}$$

$$\therefore \boxed{\frac{\Delta h_{o\,stage}}{U^2} = \frac{\Delta c_{\theta_{2,3}}}{U} = \left(1 - 2\frac{c_z}{U}\tan\alpha_2\right)}$$





- **if no exit swirl**

$$\Delta c_{\theta_{2,3}} = -c_{\theta_2}$$

$$\Rightarrow \boxed{\frac{\Delta h_{o_{stage}}}{U^2} = -1} \textbf{(V.29)} \Rightarrow \alpha_2 = -\beta_3 = \tan^{-1}\left(U/c_z\right)$$

half of power/stage vs. impulse
but usually higher efficiency

Mechanics and Thermodynamics of Propulsion, Hill and Peterson

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Turbine Degree of Reaction Summary

- Can use degree of reaction to help lead design choices
  - lower degree of reaction will tend to produce more power per stage
    - for same blade speed, stage inflow angle, stage outflow angle
  - balancing pressure drop between nozzle and rotor will tend to increase turbine efficiency

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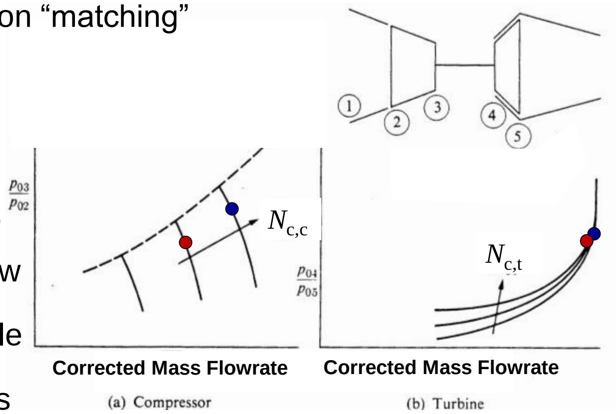
Compressor-Turbine Matching

- Need to “match” compressor, turbine on same spool
  - important part of design/operational analysis
- Steady operation “matching” considerations
  1. N (RPM)
  2. $\dot{m}_a$ vs. $\dot{m}_t$ (incl. b, f)
  3. $\dot{W}_t = \dot{W}_c$ + auxiliaries, shaft losses
- Determines how Pr_c, RPM, etc. vary with throttle setting and flight conditions



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Turbines

Stresses and Blade Cooling

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Turbine Stresses/Operational Limits

- Turbine blades experience large stresses: bending, thermal, centrifugal (rotor: 10^4-10^5 g)
- Materials exhibit significant loss of strength, enhanced creep at high T
  - low strength at modern engine T_{o4} (high ST, η_{th}) T_{o4} >1400°C (2500°F)

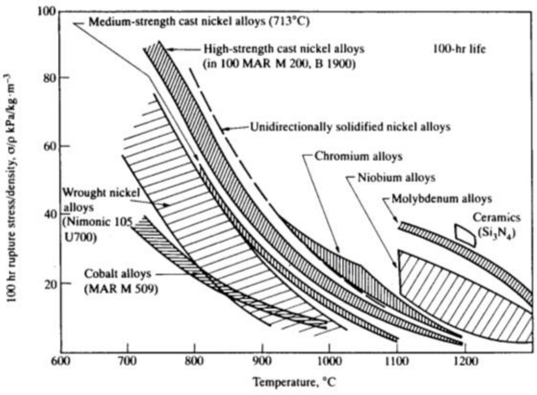

FIGURE 8.14 Variations in specific rupture strength (100 hr) with service temperature for various classes of heat-resistant materials. (Courtesy Imarigeon [10].)

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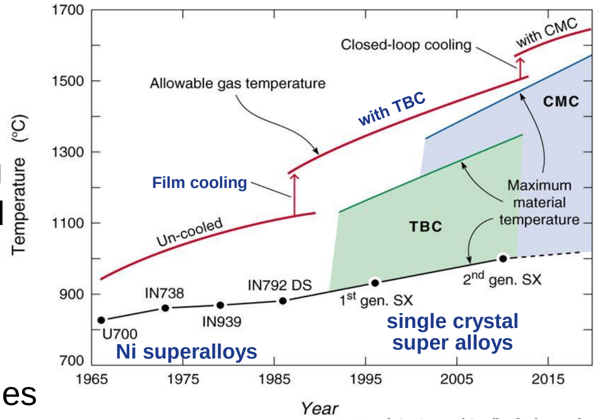
Turbine Inlet Temperature Evolution

- Solutions
  - high temperature materials
  - blade cooling
  - TBC (thermal barrier coatings)
- Cooling usually limited to 1st or few turbine stages after combustor



www.virginia.edu/ms/research/wadley/high-temp.html

to minimize amount of bleed air required

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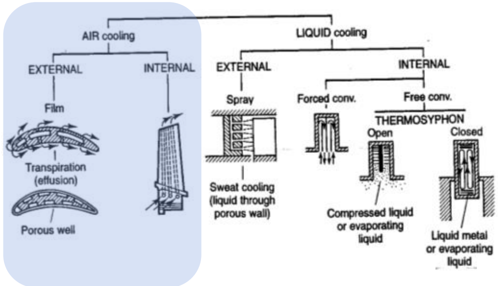
Turbine Blade Cooling

- Usually use compressor (bleed) air in aircraft engines
- Configurations
  - internal passages
  - external
    - film cooling
    - tip cooling
- Heat transfer designed to
  - focus on "hot" spots and initial stages
  - minimize stress concentration



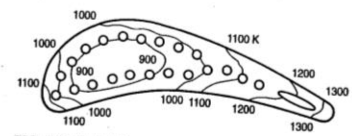


FIG. 7.35 Typical temperature distribution (from Ref. [12])

Gas Turbine Theory, Cohen, Rogers and Saravanamuttoo

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Turbine Blade Cooling

- Rotor and nozzle cooling configurations

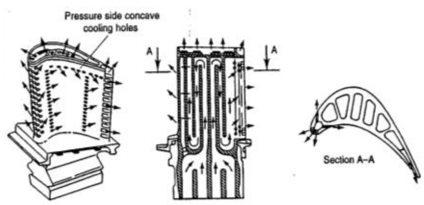

FIG. 7.30 Cooled turbine rotor blade [courtesy General Electric]

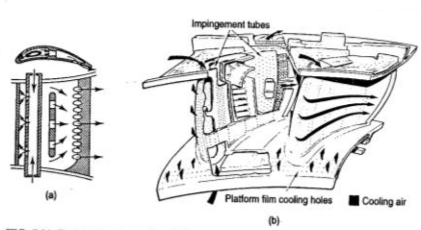

FIG. 7.31 Turbine nozzle cooling [(b) courtesy Rolls-Royce]

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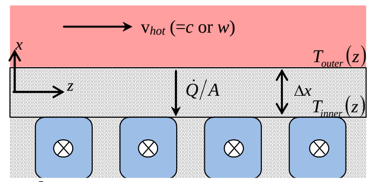
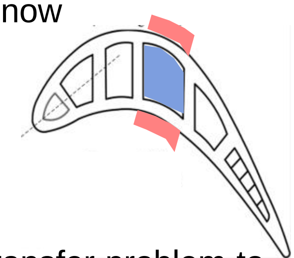
Introduction to Turbine Heat Transfer Analysis



- Consider a simplified version of a (half) turbine blade
- Inner cooling only
  - neglect film and tip cooling for now
  - hot gas (combustor products) flows over outer surface
  - "cold" gas (bleed air) flowing through inner passage(s)
  - turbine blade "wall" in between



- Will analyze this simplified heat transfer problem to understand heat transfer and blade cooling issues

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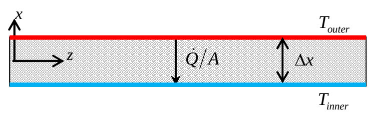
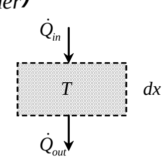
Conduction Heat Transfer



- Start with description of conduction heat transfer through the wall
  - assume one-dimensional
  - top side of wall uniform temp. (T_{outer})
  - bottom side of wall uniform temp. (T_{inner})
- Energy equation
  - differential CV $\quad \dot{Q}_{in} = \frac{d}{dt}(mcdT) + \dot{Q}_{out}$
  - steady $\quad \dot{Q}_{in} = \dot{Q}_{out}$



- Model for $\dot{Q}$
  - Fourier's Law (1d) $\quad \dot{q} \equiv \frac{\dot{Q}}{A} = -k\frac{dT}{dx}$ (Thermal Conductivity → k)

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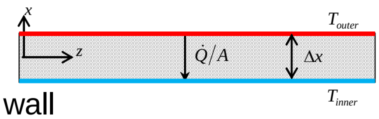
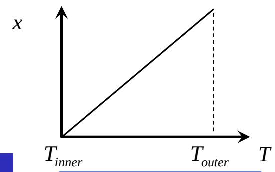
Conduction and Thermal Conductivity

- For steady, uniform material
  - T gradient is a constant $\frac{dT}{dx} = \frac{-\dot{q}}{k}$
  - so T varies linearly through wall
- Thermal conductivity
  - insulators like ceramics have much lower conductivities than metals





$$\dot{q} = -k\frac{(T_{outer} - T_{inner})}{\Delta x}$$

(V.30)

like (IV.31)

Material	k (W / mK) at 1000°C
Nickel Super Alloys	20-30
Ceramic TBC's	1-2

- so TBC will produce much lower heat flux for same temperature gradient

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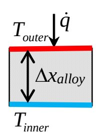
Effect of Adding TBC Coating

- Example Ni alloy with

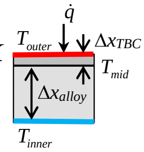
$\Delta x_{alloy} = 5mm$, $k_{alloy} = 25W/mK$, $T_{outer} = 1400K$, $T_{inner} = 1120K$



$$\dot{q} = -k\frac{dT}{dx} = -25\frac{W}{mK}\frac{-280K}{5mm} = \boxed{1400\frac{kW}{m^2}}$$

- Now add 500μm TBC

$\Delta x_{TBC} = 0.5mm$, $k_{alloy} = 1.5W/mK$

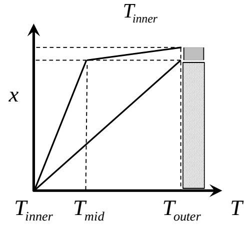


$$\dot{q}_{alloy} = \dot{q}_{TBC} \Rightarrow k_{TBC}\frac{(T_{outer} - T_{mid})}{\Delta x_{TBC}} = k_{alloy}\frac{(T_{mid} - T_{inner})}{\Delta x_{alloy}}$$

$$\Rightarrow T_{mid} = \frac{T_{outer}\,\Delta x_{alloy}/k_{alloy} + T_{inner}\,\Delta x_{TBC}/k_{TBC}}{\Delta x_{alloy}/k_{alloy} + \Delta x_{TBC}/k_{TBC}}$$

- for same T_{outer}, T_{inner}

$$\boxed{T_{mid} = 1225K} \quad \dot{q} = 25\frac{W}{mK}\frac{105K}{5mm} = \boxed{525\frac{kW}{m^2}}$$



thin coating leads to much lower metal T … much less heat transfer… and over half of ΔT occurs across thin TBC

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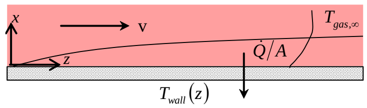
Convective Heat Transfer

- Convective heat transfer between gas flows and blade walls (including boundary layer development)
- As in rocket TCA analysis
  - $h=h(Re_z,Pr)$ ← Prandtl number
  - so T_{wall} varies downstream
  - e.g., for laminar flow over flat plate (though turbine blades are not flat, and flow not laminar)



$$\dot{q} = h\left(T_{gas,\infty} - T_{wall}\right) \quad \text{(V.31)}$$

Convective Heat Transfer Coeff.

$Pr \equiv \nu/\alpha$ $\alpha \equiv k/\rho c_p$ ← thermal diffusivity

(V.32a) Stanton Number:

$$St = \frac{h}{\rho_{g_\infty} c_{p_{g\infty}} \mathrm{v}_\infty} = 0.332 Re_z^{-1/2} Pr^{-2/3}$$

averaged over full length of plate (L)

$$\frac{\bar{h}}{\rho_{g_\infty} c_{p_{g\infty}} \mathrm{v}_\infty} = 0.664 Re_L^{-1/2} Pr^{-2/3} \quad \text{(V.32b)}$$

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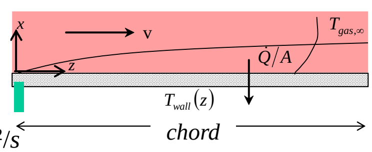
Convective Heat Transfer - External

- Example hot air
- Analysis

c=1.8cm p=12bar
T_{gas}=1700K
T_{wall}=1400K
v=350m/s
Pr=0.8 $\nu=2.55\times10^{-5} m^2/s$



$$h_{1mm} = 0.332\left(\rho_{g_\infty} c_{p_{g\infty}} \mathrm{v}_\infty\right) Pr^{-2/3} Re_{1mm}^{-1/2} \qquad \rho_{g_\infty} c_{p_{g\infty}} = \frac{p}{T}\frac{\gamma}{\gamma-1} \qquad Re_z = \frac{\mathrm{v}z}{\nu}$$

$$= 0.332\left(\frac{1200kPa}{1700K}\frac{1.28}{1.28-1}350\,m/s\right)0.8^{-0.667}\left(\frac{350\,m/s\,(0.001m)}{2.55\times10^{-5}\,m^2/s}\right)^{-0.5}$$

$$= 0.332\left(1130\frac{kW}{m^2K}\right)0.8^{-0.667}13700^{-0.5} = 3.7\frac{kW}{m^2K} \qquad \bar{h} = 2h_{x=c} = 1.8\frac{kW}{m^2K}$$

$$\dot{q}_{1mm} = 3.7\frac{kW}{m^2K}(1700-1400)K = \underline{1.1 MW/m^2} \qquad \dot{q}_{total} = \boxed{525\,kW/m^2}$$

highest heat loads close to leading edge of blades, will require more cooling

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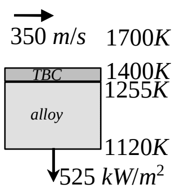
Turbine Blade Analysis

- Comparing the 2 examples
  - conduction through TBC-coated alloy =525 kW/m^2
  - convective heat transfer into blade =525 kW/m^2



- So together they could represent a single, steady-state problem

$$\dot{q}_{convection,in} = \dot{q}_{conduction,through}$$

- Next step is to investigate bleed air cooling requirement

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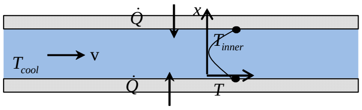
Cooling – Convection Internal Flow

- In pipe/channel flow can't assume infinite flow



  - boundary layers meet and central flow changes with axial distance
- Now $\frac{\dot{Q}}{A} = \frac{\dot{Q}}{L \times perimeter} = h\left(T_{inner} - T_{bulk,coolant}\right)$ — **Bulk avg. temp.**
- Need h expression for convection into channel
  - turbulent flow, profile still developing
  - averaged over channel length

(V.33) $$\bar{h} = 0.036\frac{k}{d} Re_d^{0.8} Pr^{1/3}\left(\frac{d}{L}\right)^{0.055}$$

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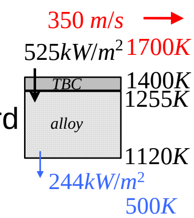
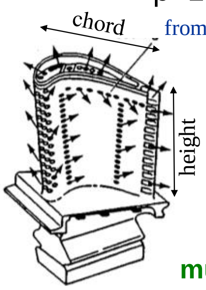
Turbine Blade Analysis

- Using information in previous examples +
  1) blade height/chord = 2
  2) 2mm sq channels spanning 80% of chord
  3) negligible spacing between channels
  4) 500K, 45 m/s bleed cooling air (Pr=0.8, p=14 bar, $\nu=2.7\times10^{-6}$ m^2/s)





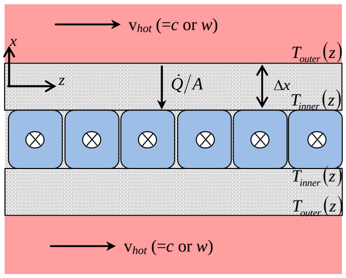
from (V.33) $h = 1.97\, kW/m^2$

$$\frac{\dot{Q}_{cool}}{A_{blade}} = \frac{0.8}{4} h\left(T_{inner} - T_{bleed}\right)$$

$$= 244\, kW/m^2$$

only ~ ½ of heat load
need to reduce heat load
or blade T will be higher

must include film cooling due to air exiting blade



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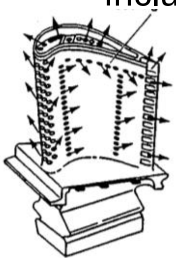
Modeling Turbine Blade Cooling

- Generally the turbine blade cooling analysis problem is approached as a simultaneous solution of the conduction and convective heat transfer issues
- Include effects of
  – accelerating flowfield of gases flowing over blades
  – non-1d geometry of blade
  – film cooling, boundary layers changes due to “blowing” from air exiting holes



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