IV. Rocket Propulsion Systems

C. Vehicle Acceleration

Accelerating Rocket

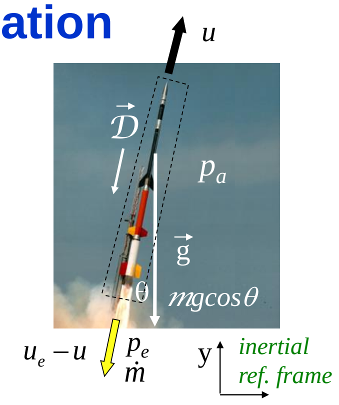
- Rocket propulsion systems are typically used to ***accelerate*** a vehicle

  $\Rightarrow du/dt \quad u \neq$ constant

- **Velocity Increment**
  - $\Delta u \equiv u_{final} - u_{initial}$
  - net increase in vehicle speed
- Lots of propellant carried on board, so mass of vehicle also changes with time

  $\Rightarrow dm/dt \quad m \neq$ constant

Rocket Acceleration



- Rocket in gravity field with thrust aligned with vehicle motion
  - momentum conservation

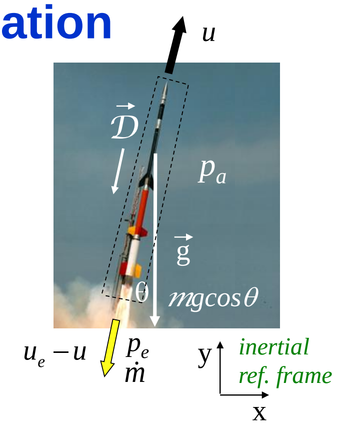
$$\vec{F}_{\substack{\text{solid body}\\ \text{on fluid}}} - \int_{open} p\hat{n}dA + \int_{open} \vec{\sigma}_{shear}dA + \int_{CV} \rho\vec{f}dV =$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{CV}\rho\vec{u}dV + \int_{CS}\rho\vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA$$

$$0-\left(-p_e+p_a\right)A_e-\mathcal{D}-mg\cos\theta=\left(\overset{-\dot{m}}{\cancel{\frac{dm}{dt}}}u+m\frac{du}{dt}\right)+\dot{m}\left(u-u_e\right)$$

$$\frac{du}{dt}=\frac{1}{m}\left[\dot{m}u_e+\left(p_e+p_a\right)A_e\right]-g\cos\theta-\mathcal{D}/m$$

Rocket Acceleration



$$\frac{du}{dt}=\frac{1}{m}\left[\underbrace{\dot{m}u_e+\left(p_e+p_a\right)A_e}_{=\dot{m}u_{eq}}\right]-g\cos\theta-\mathcal{D}/m$$

$$\frac{du}{dt}=\underset{\frac{-dm/m}{dt}}{\cancel{\frac{\dot{m}}{m}}}u_{eq}-g\cos\theta-\mathcal{D}/m$$

$$\boxed{du=-\frac{dm}{m}u_{eq}-\left[g\cos\theta+\mathcal{D}/m\right]dt} \quad \textbf{(IV.5)}$$

- Describes velocity change of rocket as function of time
  - can be integrated (with control) for vehicle trajectory

Velocity Increment

- To find Δu, integrate $du = \frac{-dm}{m} u_{eq} - [g\cos\theta + \mathcal{D}/m]dt$
- In general, all the variables can change with time
  - u_{eq}, θ, g, $\mathcal{D}$, m
  - simplest case: u_{eq} constant and $g, \mathcal{D}$ negligible

$$\int_{u_{initial}}^{u_{final}} du = \Delta u = -u_{eq} \int_{m_{initial}}^{m_{final}} \frac{dm}{m} - 0$$

Ideal Rocket Eqn.

$$\boxed{\Delta u = -u_{eq} \ln \frac{m_{final}}{m_{initial}} = u_{eq} \ln \frac{m_{inital}}{m_{final}}} \quad \textbf{(IV.6)}$$

Mass Ratio $\mathcal{R} \equiv \frac{m_o}{m_b}$ Initial mass / “Burnout” mass = m_o-m_p

$$I_{sp} = \frac{u_{eq}}{g_e} \qquad \textbf{(IV.7)} \quad \boxed{\Delta u = I_{sp} g_e \ln \mathcal{R}}$$

Ideal case
- *Δu=fn(mass change, Isp)*
- *not dependent on burn time*

Rocket Equation: Gravity and Drag Losses

- In more general case

$$\int du = \int \frac{\overset{\text{“}\tau\text{”}}{\dot{m} u_{eq}}}{m} dt - \int g\cos\theta dt - \int \frac{\mathcal{D}}{m} dt - ...other\,terms$$

$$\boxed{\Delta u = \underbrace{\Delta u_{propulsion}}_{} - \Delta u_{\substack{gravity\\loss}} - \Delta u_{\substack{drag\\loss}} - \Delta u_{steering}} \quad \textbf{(IV.8)}$$

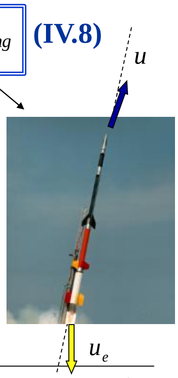
if u_{eq} still constant $u_{eq} \ln \mathcal{R}$

- comes from having to lift mass against a gravity field
- reduce by dropping m early
  - drop propellant fast (**short burn times**)
  - drop “dead” mass (**staging**)

tradeoff

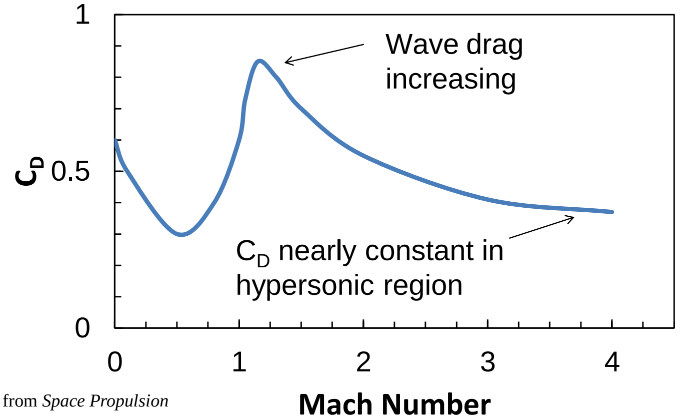
$$\mathcal{D} \sim C_D \frac{1}{2} \rho u^2 A_{ref}$$

- want low u & time where density is high (low alt.)
- lower C_D=$C_D(M$, shape)
- also want to minimize vehicle stresses



Drag Coefficient

- Example drag coefficient for a "slender" shaped rocket (also depends on AOA)



Adapted from *Space Propulsion Analysis and Design* (1995), Fig. 2.13

LEO Velocity Budgets

Vehicle	Orbit: $h_p \times h_a$ (km) inclination (deg)	Δu_{LEO}	Δu_{grav}	Δu_{drag}	$\Delta u_{steering}$	Δu_{rot}*	$\Delta u=\Sigma\Delta u_i$
Ariane A-44L	170 × 170† 70	7802	1576	135	38	–413	9138
Atlas I	149 × 607 27.4	7946	1395	110	167	–375	9243
Delta 7925	175 × 319 33.9	7842	1150	136	33	–347	8814
Space Shuttle	196 × 278 28.5	7794‡	1222	107	358	–395	9086**
Saturn V	176 × 176 28.5	7798	1534	40	243	–348	9267
Titan IV/ Centaur	157 × 463 28.6	7896	1442	156	65	–352	9207

From Table 2.10 in "Space Propulsion Analysis and Design," Humble, Henry and Larson, McGraw Hill, 1995.

Ascent varies between 8.8 and 9.3 km/s for these selected launch vehicles.

*Negative sign indicates beneficial effect of rotation.

†The third stage of Ariane 44L uses a continuous burn into a geosynch. transfer orbit; arbitrarily terminated burn at 170 km to give better comparison with other vehicles.

‡Injection occurs at 111 km.

**An additional Δu= 144 m/s is required to circularize at apogee.

from (IV.8)

$$\Delta u_{propulsion} / u_{eq} = \ln \mathcal{R}$$

$$\Delta u_{LEO} \sim 2.5\text{-}3.5 \times u_{eq,chem}$$

$$\Rightarrow \mathcal{R}_{LEO} \sim e^{2.5\text{-}3.5}$$

$$\sim 12\text{-}35$$

90-97% propellant