Steady Nozzle Flow

- What do (should) you know about CD nozzle from basic gasdynamics (nonreacting, cpg, 1-d)? e.g., vs. back pressure
  - If $p_b/p_o < p_{sub}/p_o$, choked flow (sonic throat, $M_t=1$)
  - Lowering $p_b/p_o$ moves shock from throat to exit
  - If $p_b/p_o < p_{shock}/p_o$, $M_e>1$ and isentropic flow in nozzle
  - $p_b=p_{sup}$, perfectly expanded exhaust

Supersonic Nozzle Solutions

- For isentropic (adiabatic, inviscid/reversible), no work, quasi 1-D, non-reacting, tpg and cpg
  - throat is sonic ($A_t=A^*$, $M_t=1$)
  - for given $\gamma$, $M = f(A/A_t)$,
    actual eqn. $A/A^* = f(M, \gamma)$
  - then $T/T_o, p/p_o, \rho/\rho_o ... = f(M, \gamma)$

- How does this change (if at all) when we look at a “real” gas ($c_p \neq$ constant, reacting)?
Nozzle Flow Equations

- Conservation equations
  - differential control volume

\[
\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \quad (\text{III.7}) \quad \rho u A = \text{const}
\]

- Momentum
  (inviscid)

\[dp = -\rho u du \quad (\text{III.8})\]

- Energy
  (adiab., no work)

\[dh = -u du \quad (\text{III.9}) \quad h_0 = \text{const}\]

Throat Condition

- Is \(M=1\) at throat still true?
- Review derivation
  - speed of sound
  \[a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s\]
  - if flow is isentropic
  \[dp = a^2 d\rho\]
  - combine with III.8
  \[-\rho u du = a^2 d\rho \Rightarrow \frac{d\rho}{\rho} = -\frac{u^2}{a^2} \frac{du}{u}\]
  - into III.7
  \[\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0\]
  - at throat \(dA/A=0\), so if \(du \neq 0\) (accel.)
  \[\Rightarrow M_t = 1\]
- So if flow isentropic, \(M_t=1\) still true in supersonic nozzle
  - no state equation information used in this derivation

AE 6050
Throat Condition

• Is flow still isentropic?
  – we have already assumed adiabatic, so does composition change and varying $c_p$ imply not reversible?

• If equilibrium flow
  – gas is always in equilibrium – no entropy production (gas re-equilibration rate due to collisions $>>$ rate we are trying to change properties)
  – YES reversible

• If frozen flow
  – only modes that are changing are always in equilibrium, ditto

• So only thing “new” about equilibrium/frozen nozzle flow are new state relations, e.g., $\rho = \rho(h, s)$.... $\rho/\rho_o \neq f(M, \gamma)$

General Solution Approach

• Let’s assume we have a given $T_o$ (or $h_o$) and $p_o$ (or $\rho_o$)
  – e.g., given reservoir conditions

• Options
  1. Solve differential forms of conservation equations, e.g., vs $dA$, but need to know $A/A^*$ (see below)
  2. Use mapping
     – with constraints from the conservation equations

\[
\rho u A = \text{const} \\
h_o = \text{const} \quad \text{and} \quad u = \sqrt{2(h_o - h)} \\
s = \text{const}
\]
Isentropic Mapping

- Given stag. cond.
  - and chosen \( h \)
  \[ u = \sqrt{\frac{2}{\gamma} (h_0 - h)} \]
  \[ \Rightarrow \rho, T, p, ... \]
  (state eqns.)
  \[ \rho = \rho(h, s) \]
  \[ T = T(h, \rho) \]
  \[ p = p(\rho, T) \]
  \[ \Rightarrow \frac{A}{A^*} \]
  (mass conserv)

\[ \frac{A}{A^*} = \frac{(\rho u)_{max}}{\rho u} \]

But with Q’s

\[ h = h(p, T) \]
\[ s = s(p, T) \]
\[ \rho = \rho(p, T) \]

Isentropic Nozzle Flow Summary

- So like shocks, understanding what happens to isentropic nozzle flow with “real” gas requires primarily understanding what happens to state equations
  - \( c_p \) (or \( h \)) vs \( T \) : with and without composition change
  - equilibrium and frozen flow assumption limits
Example: H₂/O₂ Rocket Nozzle

- Stagnation conditions
  - 175 atm, 3760 K
  - χ_{O₂}=29%
- 3 assumptions made
  - calorically perfect (frozen) flow, γ=1.36
  - equilibrium flow
  - chemically frozen flow
  - which is which?

Example: H₂/O₂ Rocket Nozzle

- Recombination of even “minor” (%) species can have large effect
  - e.g., on T
  - large chemical energies
Other Properties

• How do $p$, $\rho$, $u$, $M$ depend on flow assumptions?
  – depends on what you hold “constant”
  • e.g., whether you compare them at the same $T$
    or the same $A/A^*$
• For example at same $T$
  – how would pressures compare between the 3 cases?

Example: $\text{H}_2/\text{O}_2$ Rocket Nozzle

• flow gives much higher $p$
  compared to other flow assumptions
  – why?