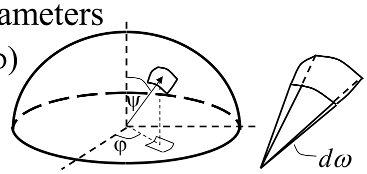
Radiation

- So far we have used statistical mechanics/statistical thermodynamics to examine equilibrium properties of matter
- Thermodynamics, being the study of matter and energy, is also applicable to pure energy, e.g., electromagnetic radiation

- As with our previous study of matter, the primary question is
  - What is the distribution of the "particles" among the possible energy levels at equilibrium?
- For radiation, the "particles" are **photons**

Radiation Field Descriptors

- Photons characterized by 2 parameters
  1) direction of propagation (ψ,φ)
     - differential 2-d angular spread given by $d\omega \equiv$ **differential solid angle**



$d\omega = \sin\psi \, d\psi d\varphi$ (units=*steradians* or *sr*)
$\int d\omega = 4\pi$

  2) frequency ν (or wavelength $\lambda = c/\nu$, c=speed of light)
     - photon energy, $\varepsilon = h\nu$
     - photon momentum, $p = h/\lambda = h\nu/c$ *NOTE: p≠pressure*
- Compare to mass particles, characterized by
  1) direction of motion
  2) speed (⇒ energy and momentum)

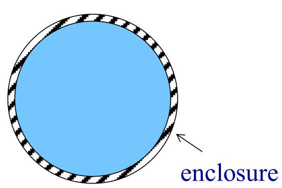
so photons are different – all have same speed (in vacuum), but different energies and momenta

Photon Distribution Function

- In general, a radiation field can be described by
  - **intensity** I, e.g., $W/m^2(/sr)$, or *sr unit often omitted*
  - **photon density** n^R, e.g., number of photons/$m^3(/sr)$
- The **photon frequency distribution function** can be written as $f^R = f^R(\nu, \omega)$ such that
  - $n^R f^R \, d\omega \, d\nu$ = number of photons per unit volume in frequency (energy) interval $\nu \to \nu + d\nu$ and given center direction within angular "cone" $d\omega$
- **Spectral density** is defined by $n^R_\nu \equiv n^R f^R$
  - number of photons/$m^3/Hz(/sr)$
- **Spectral intensity** is $I_\nu = c\, h\, \nu\, n^R_\nu$
  - $W/m^2/Hz(/sr)$

Radiative Equilibrium

- What do I_ν or n^R_ν look like if the radiation energy mode is in "internal" **equilibrium**
  - i.e., equilibrium distribution of photons among the possible energy levels
- To find this, consider a radiation field interacting with matter
  - radiation field inside (vacuum) enclosure that is opaque to radiation, i.e., nontransmitting and absorbs all incident radiation
  - since radiation can not escape, will eventually reach <u>equilibrium with enclosure</u>
  - if in equilibrium with radiation field, then body must also emit equilibrium radiation, known as a **blackbody**
- Can think of this as an equilibrium photon gas



Georgia Tech College of Engineering School of Aerospace Engineering

Approach for Equilibrium Distribution

- Use same statistical mechanics approach developed for ideal gas ("particles")
  - but photon gas follows *Bose-Einstein statistics* (can have unlimited number of bosons in a single quantum state)
- Previously presented that most probable macrostate for *B-E* statistics has distribution $N_i^* = \frac{g_i}{e^{\alpha+\beta\varepsilon_i}-1}$
  - where α was Lagrange multiplier to satisfy constraint $N = \sum N_i$, and β was to satisfy $E = \sum N_i \, \varepsilon_i$
- For photons, there is no constraint on $N \Rightarrow$ **α=0**
  - nothing prevents 1 high energy photon from becoming 2 low energy photons (no equivalent to mass conservation)
- Using Boltzmann relation (S=$k\ln\Omega$), and TD expression $(\partial S/\partial E)_{V,N} = 1/T$ again gives $\boldsymbol{\beta = 1/kT}$

Georgia Tech College of Engineering School of Aerospace Engineering

Equilibrium Frequency Distribution

- So equilibrium (most probable) photon distribution is

$$N_i^* = \frac{g_i}{e^{\varepsilon_i/kT}-1}$$

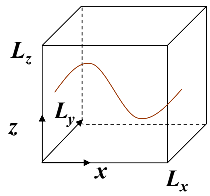
- Assuming any photon frequency (energy) is possible, i.e., the distribution is continuous, and with $\varepsilon_i = h\nu$

$$N(\nu)\, d\nu = \frac{g(\nu)}{e^{h\nu/kT}-1}$$

*from this point, will drop * notation; all results for equilibrium distributions*

- This leads to question, what is degeneracy $g(\nu)$?
- For our continuous distribution, this is equivalent to asking how many quantum states are in the frequency range ν to $\nu + d\nu$
  - this is similar to asking about extended degeneracy for translating particles in a box

Radiation Enclosure



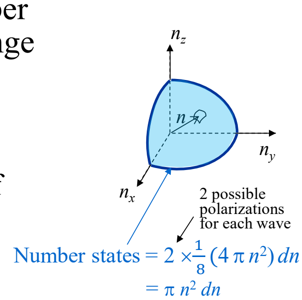
- Examine radiation enclosure using similar approach as traveling (translating) particle in a box; consider eigenvalues for **standing EM waves** in enclosure
- Translating particle in a box quantum numbers were $n_i = K_i L_i/\pi$
  - periodic boundary conditions gave $K_i = (2\pi/h)(m\text{v}_{i,n_i})$
- Recognize that $m\text{v}$ was particle momentum
  - rewriting for photons, with momentum=$h\nu/c$, and using isotropy (same result in any direction)

    $\Rightarrow$ photon quantum numbers in enclosure are $n = (2L/c)\nu$ or $n^3 = (8V/c^3)\,\nu^3$

Degeneracy

$$n^3 = (8V/c^3)\,\nu^3$$

- Recall the photon degeneracy is number of quantum states in the frequency range ν to $\nu + d\nu$
- This is related to number of quantum states in range n to $n+dn$
  - which is number of state in portion of spherical shell in "n-space" of radius n that is restricted to $n_x, n_y, n_z > 0$
  - equals $\pi n^2 dn$
- From our relation between n and ν



$$d(n^3) = (8V/c^3)\,d(\nu^3)$$

$$\Rightarrow \pi n^2 dn = (8\pi V/c^3)\,\nu^2\,d\nu = g(\nu)\,d\nu$$

Georgia Tech College of Engineering School of Aerospace Engineering

Equilibrium (Blackbody) Radiation

- So inserting this into our earlier equilibrium photon distribution function

$$N(\nu)\,d\nu = \frac{g(\nu)}{e^{h\nu/kT}-1} = \frac{8\pi V}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/kT}-1}$$

- In terms of the photon spectral density

$$n_\nu^R = \frac{N(\nu)}{4\pi V} = \frac{2\nu^2/c^3}{e^{h\nu/kT}-1}$$

4π steradians (isotropic)

- In terms of spectral intensity

$$I_{\nu,b} = h\nu c\, n_\nu^R = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$$

Planck added this after postulating quantized radiated energy, so able to predict long λ radiation behavior

- *Planck Function*
- *Planck's Law*
- ***Planck Blackbody Distribution**, so often denoted $I_{\nu,b}$*

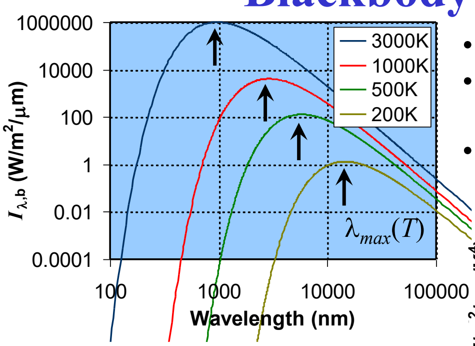
– in terms of wavelength

$I_\nu = dI/d\nu$, $\lambda = c/\nu$, $d\lambda/\lambda = d\nu/\nu$

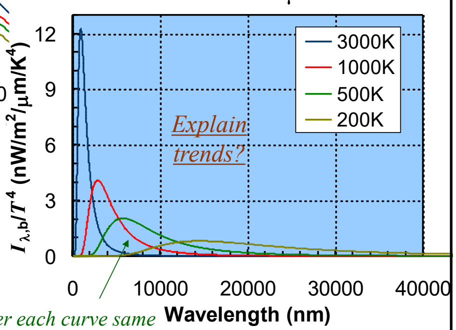
$$I_{\lambda,b} = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT}-1}$$

Georgia Tech College of Engineering School of Aerospace Engineering

Blackbody Intensity



- $I_{\lambda,b}$ ↑ with T for all λ
- λ_{max} shifts blue as T↑
  – $\lambda_{max}T$ = 2897.8 μm K
- $I_{\lambda_{max},b} / T^5$ = constant ~0.004 nW/m²/μm/K⁵



Area under each curve same

- Spectrally integrated I_b = fn(T)
  – I_b/T^4~18 nW/m²/K⁴ ≡ σ/π *(Stefan-Boltzmann constant)*
  – so total radiation intensity (energy) scales as T^4

Georgia Tech | College of Engineering | School of Aerospace Engineering

Photon Distribution Function

- In terms of the distribution function

$$f^R(\nu) = \frac{I_\nu}{I_{tot}} = \frac{n_\nu^R}{n^R} = \frac{\pi}{\sigma T^4}\frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$$

- So like matter, we can describe a radiation field, **that is in equilibrium**, with a single parameter, its temperature
  - the temperature is a measure of how the "particles" (photons) are distributed among the energy levels
- Unlike many of the energy modes of matter, radiation is often far from being in equilibrium
  - especially the radiation fields produced by gases at typical pressures

Georgia Tech | College of Engineering | School of Aerospace Engineering

Equilibrium Radiation and Energy

- Examine total equilibrium radiation intensity as function of temperature, $I_b(T)$
- What is the total energy of an atomic gas (neglecting electronic energy)?
  - $E = 3/2\ NkT$
    $= 3/2\ pV$
- How long to deplete all this energy due to blackbody radiation?
  - $t = 3/2\ p(V/A)/I_b$
  - for $1mm$ diameter spherical region at $1bar$