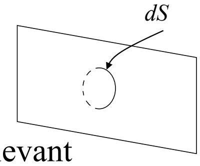
Molecular Fluxes

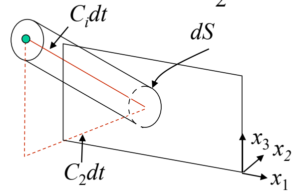
- To perform more rigorous derivation of pressure in terms of c_i and $f(c_i)$, we will need to examine fluxes of molecular properties across some differential area in some arbitrary plane in space
  - number of gas molecules per unit time passing through a differential area (dS) *that moves with the flow*
  - so only random component (C_i) is relevant



- There are many different gas properties, so we can look at many different fluxes
  - begin by finding flux of molecules with a specific velocity, i.e., molecules of **class** $C_i dV_C$

One-Way Differential Number Flux

- We want to find the number of gas molecules (of our chosen "class") passing through dS per unit time
- For our exercise, we will pick a plane normal to x_2
- Molecules with initial velocity C_i will only pass through dS in dt if they starts in a limited volume
  - slant cylinder of length $C_i dt$ and height $C_2 dt$



assumes $C_i dt << \lambda$, so no collisions to worry about, but $dt \to 0$, so okay

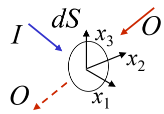
cyl. volume; number density of molec. with c_i; total gas number density

$$\text{Differential Number Flux} = \frac{\overbrace{C_2\, dt\, dS}^{\text{cyl. volume}} \times \overbrace{nf(C_i)dV_c}^{\text{number density of molec. with } c_i}}{dS\, dt} = nC_2\, f(C_i)dV_C$$

1-D Momentum Flux

- Recall from mechanical viewpoint (Newton's Law), pressure (normal stress) is change in normal (1-way) momentum flux of molecules
  - examine incoming and outgoing mom. flux through dS

$$p_2 = \text{rightward flux} + \text{leftward flux}$$



$$p_2 = \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} \underbrace{\left[m\left|C_2\right|\right]}_{\text{Mom/molec}} \underbrace{n\left|C_2\right| f\left(C_i\right)}_{\text{number flux}} dC_1 dC_2 dC_3$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} \left[m\left|C_2\right|\right] n\left|C_2\right| f\left(C_i\right) dC_1 dC_2 dC_3$$

$$p_2 = nm\int_{-\infty}^{\infty} C_2^2\, f\left(C_i\right) dV_C \equiv nm\overline{C_2^2}$$

normal stress = transport of normal momentum by random molec. flux in same direction

Pressure

- Define **equilibrium** pressure to be average of normal stress (momentum flux) in all 3 directions

$$p = \frac{1}{3}\left(p_1 + p_2 + p_3\right) = \frac{1}{3} nm \underbrace{\left(\overline{C_1^2} + \overline{C_2^2} + \overline{C_3^2}\right)}_{\equiv \overline{C^2}}$$

$$\boxed{p = \frac{1}{3} nm\overline{C^2}}$$

not valid in flows with strong gradients (⇒ nonequilibrium)

  - so in equilibrium $\overline{C^2} = 3\overline{C_2^2}$
    $p = p_2 = p_1 = p_3$

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Other Transport Processes: Diffusion

- Turns out if we revisit transport of other properties by molecular motion, they are also due to one-way fluxes of those properties
- Energy transport in *j*-direction due to random molecular motion: **"heat" diffusion**

$$q_j = \int_{-\infty}^{\infty} \left(\frac{1}{2} m C^2\right) n C_j f(C_i) dV_c = \frac{1}{2} \rho \overline{C_j C^2}$$

- Transverse momentum transport in *j*-direction due to random molecular motion: **shear stress**

$$\tau_{ij} = \int_{-\infty}^{\infty} (m C_i) n C_j f(C_i) dV_c = -\rho \overline{C_i C_j}$$

no assumptions about molecular force models (so not just true for "elastic" spheres) ***EXCEPT*** *molecular forces exerted only over small distances (<<λ), i.e.,* ***perfect gas assumption***