

Entropic State Equation

- To find entropy change between 2 states of simple compressible substance without composition change, as a function of T, p

$$dS = \left. \frac{\partial S}{\partial T} \right|_p dT + \left. \frac{\partial S}{\partial p} \right|_T dp$$

$$\left. \frac{1}{T} \frac{\partial H}{\partial T} \right|_p = \frac{C_p}{T}$$

$$\left. -\frac{\partial V}{\partial T} \right|_p = -\alpha V$$

$$dS = C_p dT/T - \alpha V dp$$

- Integrating from 1→2
- Can follow similar procedure to show

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{C_p}{T} dT - \int_{p_1}^{p_2} \alpha V dp$$

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{C_v}{T} dT + \int_{V_1}^{V_2} \frac{\alpha}{\kappa} dV$$

Enthalpic State Equation

- To find enthalpy change between 2 states of simple compressible substance without composition change, as a function of T, p
 - from H def'n. $dH = dU + pdV + Vdp$
 - from Gibb's eqn. $dU = TdS - pdV$
 - combine $dH = TdS + Vdp$
 - but $dS = C_p dT/T - \alpha V dp$ $dH = C_p dT - \alpha VT dp + Vdp$
 $dH = C_p dT + (1 - \alpha T) Vdp$
- Integrating from 1→2

$$H_2 - H_1 = \int_{T_1}^{T_2} C_p dT + \int_{p_1}^{p_2} (1 - \alpha T) V dp$$

Internal Energy State Equation

- To find energy change between 2 states of simple compressible substance without composition change, as a function of T, p

– $U = H + PV$

$$dU = dH - Vdp - pdV$$

– $dH = C_p dT + Vdp - \alpha TVdp$

$$dU = C_p dT - \alpha TVdp - pdV$$

– also $dV = \alpha VdT - \kappa Vdp$

so

$$dU = C_p dT - \alpha TVdp - p(\alpha VdT - \kappa Vdp)$$

– then

$$dU = (C_p - \alpha pV) dT - (\alpha T - \kappa p) Vdp$$

- Integrating from 1→2

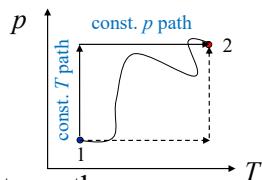
$$U_2 - U_1 = \int_{T_1}^{T_2} (C_p - \alpha pV) dT - \int_{p_1}^{p_2} (\alpha T - \kappa p) Vdp$$

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State Equations

- These expressions employ two integrals
 - one over T , one over p
- So is each integrand solely a function of one variable?
 - no
 - we can choose any path to calculate state variable change if we know initial and final states
 - e.g., constant T path then constant p path
 - need $c_p(T)$, $\alpha(T)$, $V(T)$ at fixed p (along constant p path)
 - need $\alpha(p)$, $\kappa(p)$, $V(p)$ at fixed T (along constant T path)



Property Changes from State Equations -4
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