

Independent Variations of TD Properties

- **Motivation**
 - Recall, a TD state is a unique (thermodynamic) description of a substance
 - identified by state variables
 - e.g., p, T, ρ, \dots intensive; m, V, \dots extensive)
 - How many TD properties must be known/specified to uniquely define the state of a system?
- **Empirical evidence** (experience) suggests
 - number of ways one can independently vary the energy of a given substance gives the number of independent TD properties

Reversible Work Modes

- What are independent ways to transfer energy?
 - Reversible work modes + Heat transfer
- What are reversible work modes?
 - work mode is reversible if
$$\delta W \equiv F dx$$
 - force F is independent of direction and rate of change of process
 - or
 - equivalent amount of energy transferred to system when x is increased by dx will be exactly the same amount of energy transferred from system when x is decreased by dx

Examples of Rev. Work Modes

Mode	dW (+ into system)
Fluid Compression	$-pdV$
Polarization	$\vec{E} \cdot d(v\vec{P})$
Liquid Surface Extension	σdA

- **Simple substance**
⇒ only one reversible work mode
- **Simple compressible substance**
⇒ only rev. work mode is fluid compression
- Also, effect of any irreversible work modes can always be accomplished by combination of reversible work + heat transfer

State Postulate

- Formal statement
 - “The number of independent, intensive TD properties of a *specified* substance is equal to the number of *relevant* reversible work modes (**n**) plus one.”
 - *specified* substance ⇒ must know fraction of each constituent in a mixture of substances (phases,...)
 - *relevant* modes ⇒ only care about reversible work modes for system in question
- Does not identify which set of $n+1$ properties are independent
- Relationships between TD properties given by **equations of state**
 - generally for intensive TD variable, $I_{n+2} \equiv I_{n+2}(I_1, I_2, \dots, I_{n+1})$
 - e.g., $p=p(\rho, T)$

Example: Simple Compressible Mixture

- Assume mixture of k pure (single-phase) substances
 - how many properties are independent?
- To *specify* substance, we need $k-1$ mole fractions χ_i
 - $\chi_i \equiv n_i/n_{tot}$ where n_i is the number of moles of pure substance i (n_i is extensive, χ_i is intensive)
 - $\sum_{i=1}^k n_i = n_{tot}; \sum_{i=1}^k \chi_i = 1$
 - so only $k-1$ mole fractions are independent

Example (con't)

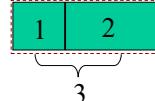
- In addition to $k-1$ mole fractions, state postulate says we need $n+1=2$ additional independent intensive variables to define a state ($n=1$ for simple compressible substance)
 - $I_3 \equiv I_3(I_1, I_2, \chi_1, \dots, \chi_{k-1})$
- To determine any *extensive* property (E) of the system, we must add one additional *extensive* property (e.g., mass) to the list
 - $E_J \equiv E_J(E_1, I_1, I_2, \chi_1, \dots, \chi_{k-1})$

Extensive and Intensive Props.

- We can create intensive properties from extensive properties
- For example, given two systems in TD equilibrium
 $\Rightarrow I_J^{(1)}=I_J^{(2)}=I_J^{(3)}$
 - by additive property of extensive variables

$$E_J^{(3)} = f(m^{(3)}, I_1, I_2) = E_J^{(1)} + E_J^{(2)}$$

$$= f(m^{(1)}, I_1, I_2) + f(m^{(2)}, I_1, I_2) \quad m = \text{mass}$$



- but if this is true, then we must be able to write

$$f(m, I_1, I_2) = mg(I_1, I_2)$$

$$\therefore E_J/m = g(I_1, I_2) \quad \text{a (mass) specific property}$$

Specific Properties

$$\frac{E_J}{m} = g(I_1, I_2)$$

- Are intensive because they are functions ONLY of intensive properties
- Examples
 - $h \equiv H/m$ specific enthalpy
 - $v \equiv V/m$ specific volume
- Can define other types of specific props.
 - e.g., H/V volume specific enthalpy
 - in general, ratios of exten. props. are intensive