Velocity Distribution Function

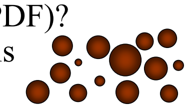
- In this section, we want to look at a rigorous derivation of equilibrium gas properties using kinetic theory
- Previously we noted that molecules have a distribution of speeds (or more properly velocities)
  - what is this function?
- Begin using a statistical approach to describe the probability that a molecule in a gas will have a specific velocity (within some small range)

$$f(\vec{c}) = f(c_1, c_2, c_3) = f(c_i)$$

Velocity (probability) Distribution Function

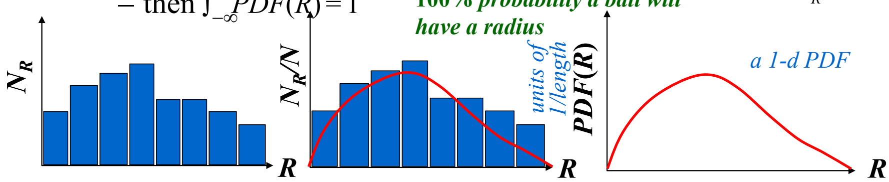
Probability Distribution Function

- What is a probability distribution function (PDF)?
- Say we have a large number of spherical balls with different sizes (radius, R)



  - could create a histogram of the number of balls (N_R) with R in some binned ranges (e.g., $R-\Delta<R<R+\Delta$)
  - or the fraction of balls in each bin
- What happens if we make the bins infinitesimally narrow?
  - get a smooth, continuous function
- This function becomes a PDF if we let $PDF(R)dR = \frac{dN_R}{N_R}$
  - then $\int_{-\infty}^{\infty} PDF(R) = 1$

100% probability a ball will have a radius



Velocity Distribution Function

- So our velocity PDF $f(\vec{c}) = f(c_1, c_2, c_3) = f(c_i)$
  - is probability of finding particle with velocity in range $(c_1, c_2, c_3) \rightarrow (c_1+dc_1, c_2+dc_2, c_3+dc_3)$
  - and

units of $1/speed^3$

$$\frac{dN_{c_i}}{N} = f(c_i)dc_1dc_2dc_3 = f(c_i)dV_c$$

$$\int\int\int_{-\infty}^{\infty} f(c_i)dc_1dc_2dc_3 = \int_{-\infty}^{\infty} f(c_i)dV_c = 1$$

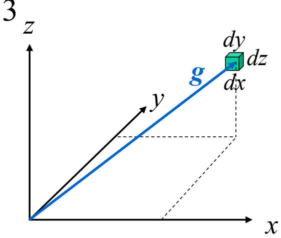
- **Average Properties**
  - to find mean (or expected) value of some property/ function that depends on molecule's velocity, i.e., $Q(c_i)$

$$\overline{Q} = \frac{\int_{-\infty}^{\infty} Nf(c_i)Q(c_i)dV_c}{N} = \int_{-\infty}^{\infty} Q(c_i)f(c_i)dV_c$$

Velocity "Space"

- Common to divide physical space into 3 (Cartesian) coordinates (x,y,z)
  - and perform integrations over differential volumes in this space, e.g.

$$\int\int\int g(x,y,z)\ dx\ dy\ dz$$



- We can also think of the possible velocities of molecules as constituting a 3-dimensional **velocity "space"**
  - can talk about how molecule "moves" through velocity space (i.e., accelerates)
  - perform integrations over velocity space, e.g.

$$\int\int\int f(c_i)\ dV_c$$